Archaeoastronomy, no. 5 (*JHA*, xiv (1983))

A REASSESSMENT OF THE HIGH PRECISION MEGALITHIC LUNAR SIGHTLINES, 2: FORESIGHTS AND THE PROBLEM OF SELECTION

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Following on from our reassessment of backsights and indicators in Part One of this paper (*Archaeoastronomy*, 4, S21–40), we attempt in Section 5 to clarify the data on foresight declinations from the forty-four putative high precision lunar sightlines. Included in this discussion is a consideration of the inherent uncertainties in these declinations due to uncertainties in the exact observing position. Discussion of the nature of the foresights themselves is presented in Section 6, leading on to the question of their selection from amongst other horizon features equally plausible as foresights *per se*; that is, without regard for the astronomical possibilities. In Section 7 we reconsider in general terms the analysis of measured declinations within the "lunar bands" around the mean standstills. The various threads from both parts of the paper are drawn together in Section 8 for a statistical reappraisal of the forty-four sightlines. General conclusions appear in Section 9.

5. *Indicated Declinations*

The main purpose of this section is to clarify the data on measured declinations at the forty-four putative sightlines; that is, to tackle question (2) of Section 1 with regard to foresights.

In columns 2–4 of Table II we list for each sightline the azimuth and altitude of the horizon foresight given by the Thoms, together with a reference to the measurements quoted. In order to facilitate later discussion of possible selection effects, we calculate in column 5 a mean lunar declination from the Thoms' quoted azimuth and altitude, assuming uniformly a refraction correction[89] appropriate to 10°C and 1005 mb pressure, and a parallax correction corresponding to a horizontal lunar parallax of 56′·9.[90] This is done in preference to quoting the Thoms' own listed declinations, since, especially in their later publications,[91] these have been calculated using the particular corrections appropriate to the lunar event assumed to have been observed; in other words, the assumed function of any sightline is already implicit in the indicated declination quoted, a fact which could be misleading in any statistical analysis of the astronomical significance of the declinations. In order to avoid this pitfall, we shall consider corrections to the mean declinations separately at a later stage, in Section 7.

The Thoms' azimuths are usually quoted accurate to 1′, and altitudes to 0′·1. Where more accurate azimuths are quoted by the Thoms, for example at Kilmartin (Temple Wood)[92] and in some of the lines at Brogar,[93] we use the more accurate value in calculating the declination despite not quoting it in Table II. Since an accuracy of 1′ in azimuth only justifies an accuracy of around 0′·4 in declination at minor standstill and around 0′·3 at major standstill (although this can be reduced to 0′·1 and less in the case of lines very near to due south),[94] we feel that in general 0′·2 is the greatest accuracy justified by the data as presented, and quote declinations to this accuracy.

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The quoted accuracy of the Thoms' altitudes may be misleading in a number of cases, for seventeen out of thirty-eight altitudes quoted accurate to 0′·1 have the decimal equal to zero. Eight of these appear elsewhere[95] quoted only to the nearest minute, calling into question the accuracy of each of the other nine. This would introduce a possible error of up to ±0′·4 in declination in each case.

The sightlines have been resurveyed by the author wherever the weather permitted. The instrument used for surveys during 1979 and 1981 was a Kern DKM-1 lightweight microptic theodolite reading to 10″. During each season, over three months of continual use on this and other projects enabled theodolite adjustment errors to be regularly monitored. The two sites in the Callanish area (Lines 14 and 15), which had been surveyed by the author and his colleagues in 1975,[96] were resurveyed and checked.

Plate bearing zero (PBZ) is the true azimuth of the zero graduation on the horizontal circle of the theodolite. When weather permitted, this was determined from observations of the Sun, timed using a calibrated quartz crystal wristwatch or digital timer. A series of twelve observations, three on each combination of solar limb and theodolite face, typically gave PBZ to ±10″. In other cases PBZ was determined from sightings of three or more Ordnance Survey triangulation stations, or, in the case of the Callanish area sites, those of the Glasgow University survey.[97] At Brogar and Airigh nam Bidearan both solar and triangulation methods were available, providing consistency checks. Measured azimuths are quoted in Table II accurate to 0′·5 and altitudes to 0′·2. Deduced declinations are also quoted accurate to 0′·2.

In practice it is not always possible or convenient to place the theodolite at the exact observing position, and a parallax correction has to be applied in order to transform a measured horizon profile into that seen from the latter. Where the Thoms have not stated that a sizeable correction has been applied, their azimuths should be reliable to within 1′ and their survey is classified 'A' in column 6 of Table II. Where our parallax correction amounted to at most about 5′, our azimuths should be reliable to within 0′·5 and our survey is classified 'A' in column 12.

In a number of cases larger parallax corrections were necessary in order to transform a measured profile into the hypothetical observed one. This introduces the possibility of some error into the quoted azimuths, because of perspective effects due to profiles not being two-dimensional, because of notches formed by junctions between hills at different distances closing or opening up, and because it is not always possible to determine sufficiently accurately from maps the distance of the foresight, which is necessary for the calculation to be reliable. The status of such surveys is given as 'B', with details of the nature of the correction given in the comment columns.

Finally, where profiles could not be surveyed at all, some were constructed by the Thoms from Ordnance Survey maps, and are classified 'C'.

An overall mean declination is estimated in column 14 to the nearest 0′·2 on the basis of the Thoms' and our own surveys. Where one of the two is of more reliable status, the value is taken from this, and where they are the same, a mean value is taken. If the mean falls between two gradations of 0′·2 then the figure nearer the Thoms' declination is taken. A few special cases are elaborated

TABLE II. Summarized details of indicated declinations, from measurements by the Thoms and the author, together with estimates by the author of inherent uncertainties in these values.

Column headings

1 Sightline number
2 Reference to measurements quoted by the Thoms
3 Azimuth quoted by the Thoms
4 Altitude quoted by the Thoms
5 Lunar declination deduced by the author from these
6 Status of the Thoms' survey (see text)
7 Comments
8 Year(s) of resurvey by the author
9 Azimuth measured by the author
10 Altitude measured by the author
11 Lunar declination deduced from these
12 Status of the author's survey (see text)
13 Comments
14 Deduced value of most probable indicated declination
15 Difference from value deduced from the Thoms' measurements alone
16 Overall status of declination determination
17 Comments
18 Assumed observing position
19 Distance of foresight in km.
20 Inherent uncertainty in azimuth
21 Inherent uncertainty in altitude
22 Deduced uncertainty in declination
23 Comments

Key to column 7 (comments)

a Measurements from the centre of the ring and *J* have been reduced to the observing position (OP).[103]

b Apparently this profile was not measured directly from the OP, for the Thoms' profile diagram[104] gives declinations as measured from the centre of the ring, and the Comet Stone and mound *M* are nowhere mentioned as theodolite positions for any profile.

d Profile constructed from the 1″ Ordnance Survey.[105]

e Profile constructed from the 6″ Ordnance Survey.[105]

f Measurements from high ground behind the site have been reduced to the OP.[106]

g Measurements have been reduced to the OP from a point some 1·5 km in front of it, 100 m off line to the right, and 6 m above it.[107]

h "The profile may be inaccurate by a minute."[108]

i Measurements from a short distance towards the foresight have been reduced to the OP.[109]

j Profile constructed from the 1″ Ordnance Survey, with check points measured using a less accurate lightweight theodolite.[106]

k Profile constructed from the 1″ Ordnance Survey of Ireland.[110]

Key to column 13 (comments)

l Measurements from the centre of the ring, the Comet Stone and L_2 have been reduced to the OP.

m Measurements from mid-way along the alignment,[111] that is about 20 m in front of the OP, have been reduced to the OP.

n Measurements have been reduced to the OP from a point some 0·9 km behind it, 200 m off line to the left, and 57 m above it (on a hill slope at 835 984).

o Trees now obscure direct view of the foresight from the OP. Measurements have been reduced to the OP from a point some 250 m behind it and 20 m off line to the right.

p Measurements from Thom's Stone *S* and the fallen menhir *M*[112] have been reduced to the OP.

Key to column 16 (overall status of declination determination)

A Measured from at or near the OP and considered reliable to 1′.

B Measurements from elsewhere reduced to the OP, so errors of up to (say) ±3′ are possible.

C No direct measurements; constructed profile only. Larger errors may be possible.

Key to column 17 (comments)

q The azimuth and altitude discrepancy here may be due to the Thoms quoting these for the top rather than the bottom of the Mid Hill "step";[104] our figures for the top of the step are Az = 135° 7′·5, Alt = 2° 8′·8, in agreement with the Thoms'. This makes little or no difference to the declination.

r The exact points on this profile measured by the Thoms[113] could not be identified, which may partly explain the azimuth and altitude discrepancy here.

s The azimuth discrepancy here is well beyond the possible error due to uncertainty in the OP. Our values are, however, in good agreement with new values quoted by the Thoms elsewhere.[95] While the Thoms also state that the profile has been remeasured,[114] the earlier values have been used in their analyses of the forty-two lines.[115] We assume our measurements to be correct here.

t Measurements by Patrick[102] are also given equal weight in determining the most probable declination value. These are

Line	Quoted azimuth	Quoted altitude	Deduced declination
20	316° 59′·5	4° 38′·6	28° 57′·4
21	317° 13′·5	4° 38′·2	29° 3′·0
22	317° 56′·0	4° 37′·2	29° 20′·2

Discrepancies between Patrick's measurements and Thom's appear largely to be due to Thom, like us, having taken measurements from an assumed OP behind the various backsights (although, unlike us, he does not state clearly where these assumed OPs are), whereas Patrick calculated values from the backsights themselves (for example, menhir S_1). The values we have adopted for OPs 2 m behind the backsights are

Line	Adopted azimuth	Adopted altitude	Deduced declination
20	316° 59′·0	4° 38′·4	28° 56′·8
21	317° 13′·0	4° 37′·8	29° 2′·4
22	317° 55′·5	4° 36′·8	29° 19′·8

u Since Thom's survey, Forestry Commission trees have filled the Bellanoch Hill notch, which now appears rounded.[116] This almost certainly largely explains the azimuth and altitude discrepancies here. We assume Thom's measurements to be correct here.

v Discrepancies here may largely be due to our having taken as foresight the point on the slope of Crackaig Hill showing the lowest declination whereas Thom took an apparently arbitrary point up the hill to the left towards (but not at) the shoulder.[117] Our values are, however, in reasonable agreement with values quoted by the Thoms elsewhere.[95] We assume our values to be correct here.

w The azimuth discrepancy here is well beyond the possible error due to uncertainty in the OP. During our survey of this site, measurements of three triangulation points (two Ordnance Survey triangulation stations and the menhir at Ardpatrick, Knapdale, whose position had been determined previously) gave an independent check of PBZ agree ingwith the results of 14 Sun azimuth observations to within 0′·25. Measurements of the foresight were repeated on both theodolite faces and checked against measurements of other horizon points shown on a 400 mm photograph. Thus we consider gross error in our own measurements unlikely and have assumed these to be correct here.

x A notch is actually formed lower down the slope of Cnoc Moy than Thom's A_2, at its junction with a hill behind, shown dotted in Thom's diagram.[118] Above this notch, in the vicinity of A_2, Cnoc Moy slopes up without any part parallel to declination lines.

y Since Thom's survey here was an early one, and his profile diagram[119] misses features of prominence equal to those shown (note the small number of surveyed points), we consider that the discrepancies here are most likely to reflect shortcomings in the earlier survey, and adopt our own measurements as correct.

Key to column 18 (assumed observing position)

- A 2 m behind back end of alignment
- Am 2 m behind back end of alignment, mid-way between menhirs S_2 and S_3[120]
- C Circle centre, as determined from present positions of stones
- M 2 m behind menhir (whether oriented towards foresight or not) or stone or stone socket
- Q 2 m behind back of group of small stones
- R Centre of ridge behind site
- S Centre of the site
- T Centre (present level) of cairn or mound
- Tb 2 m behind back of cairn or mound
- Tf Immediately in front of cairn or mound
- Ts Immediately by side of cairn or mound
- X Unmarked and deduced from other features at the site (see Section 3)
- Z 2 m behind back end of alignment, mid-way between the northern stones of the two sides of the avenue (Stones 8 and 19).[121] As noted in Section 3, the foresight cannot actually be seen from here, so the declination quoted in column 14 must be regarded as a hypothetical value for comparison with Thom's value. Declinations which could actually have been observed from nearby positions are included within the uncertainty limits quoted in column 22.

Key to column 23 (comments)

- B Uncertainties taken as 1 m either side of assumed OP, and 0·5 m vertically up or down.
- D Additional uncertainty due to unknown distance of OP behind back end of alignment; 0–10 m allowed for.
- E Uncertainties correspond to the estimated original extent of the cairn or mound, allowing for observations from the top at original height, or the front or sides at present ground level. See Section 3 for the present extent of the cairn or mound.
- F The indication here unambiguously specifies the NE side of mound *M* as the OP (see Section 3), so uncertainties due to the extent of the mound are ignored.
- G Additional uncertainty taken corresponding to the difference between the OP as determined by the Thoms and by the author (see Section 3).
- H OP could be anywhere on the ridge used for postulated observations to the nearby notch to the north;[122] uncertainties taken corresponding to this.
- J Uncertainties taken corresponding to movement to nearby observing positions from which the foresight could actually have been seen, such as a few metres to the west of the avenue and upon the rocky outcrop to the south of the site (see Section 3).
- K Additional azimuth uncertainty due to the uncertainty in the exact orientation of the stone alignment; this enters at this site because there is no actual foresight here, only an arbitrary point on a featureless horizon.
- L Since there is no particular candidate for OP at this site, uncertainties are taken corresponding to movement to any position within or just behind the site.
- N Additional azimuth uncertainty taken corresponding to observations from S_2 or S_3 themselves rather than their mid-point.
- P Because of the uncertainty in the original circle position (see Section 3), the assumed uncertainty in the OP is 10 m in any direction.
- R Additional altitude uncertainty due to the possibility of the OP being within, as well as behind, the group *Q*.

1	2	3	4	5	6	7	8	9	10	11	12	13
		° ′	° ′	° ′				° ′	° ′	° ′		
1	*MRBB*, 166	192 01	−0 21·7	−29 09·8	A		81	192 01·0	−0 21·2	−29 09·2	A	
2	*MRBB*, 134	227 50	1 05·2	−18 43·8	A		79	227 49·5	1 05·8	−18 43·4	B	l
3	,,	227 36	1 04·6	−18 50·0	B	a	79	227 35·5	1 05·8	−18 49·0	B	l
4	,,	227 13	1 05·0	−18 58·8	B	a	79	227 13·0	1 06·0	−18 57·8	B	l
5	,,	135 08	2 08·8	−18 49·4	B	b	79	135 06·0	2 08·0	−18 49·6	A	
6	,,	133 28	2 00·1	−18 19·2	B	b	79	133 27·5	2 01·4	−18 17·6	B	l
7	,,	24 40	1 01·5	29 25·6	A		79	24 33·5	1 00·2	29 26·0	B	l
8	,,	24 28	0 58·4	29 25·4	A		79	24 23·5	0 57·2	29 25·0	A	
9	,,	336 23	0 15	28 48·4	A		79	336 24·0	0 14·6	28 48·2	A	
10	,,	336 47	0 14	28 52·8	A		79	336 47·0	0 13·2	28 52·0	B	l
11	*MRBB*, 130	307 35	0 27·8	19 08·6	A		79	307 35·5	0 27·6	19 08·6	A	
12	*MLO*, 76	158 30	−0 13·0	−29 08·8	A		—					
13	,,	160 05	−0 11·0	−29 26·6	A		—					
14	,,	188 45	1 35·8	−29 11·6	C	d	79, 75	188 44·5	1 34·4	−29 13·0	A	
15	,,	162 40	1 06·5	−28 35·4	C	e	81, 75	162 42·5	1 06·8	−28 35·6	A	
16	,,	197 57	1 58·0	−28 54·0	A		79	197 50	1 54	−29 00	B	m
17	*MRBB*, 174	30 55	0 32	29 17·4	A		79	30 46·0	0 32·2	29 20·4	A	
18	*MLO*, 76	126 49	0 08·0	−19 00·0	B	f	—					
19	,,	233 23	0 41·5	−18 20·2	A		81	233 25·0	0 42·8	−18 18·2	A	
20	,,	316 59	4 37·7	28 56·4	A		81, 79	316 58·0	4 38·4	28 56·4	A	
21	,,	317 13	4 37·7	29 02·2	A		81, 79	317 10·5	4 37·8	29 01·4	A	
22	,,	317 53	4 37·2	29 18·8	A		81, 79	317 55·0	4 36·8	29 19·4	A	
23	,,	207 56	0 18	−28 48·4	B	g	81	207 48	0 25	−28 43	B	n
24	,,	321 30	2 56·0	29 13·4	A		79	321 29·5	2 58·6	29 15·4	B	o
25	,,	322 13	2 54·0	29 28·4	A		79	322 11·0	2 57·6	29 31·0	B	o
26	,,	324 59	1 38·0	29 17·2	A		81	324 58·0	1 39·0	29 17·8	A	
27	,,	204 00	0 55·0	−29 26·2	A		79	204 06·0	0 52·2	−29 27·6	A	
28	*MRBB*, 170	328 49	0 11·2	29 17·2	A		79	328 49·0	0 10·4	29 16·2	A	
29	,,	327 28	0 44·2	29 26·0	A		79	327 28·5	0 44·2	29 26·2	A	
30	*MLO*, 76	326 00	0 45·0	28 54·0	B	h	81, 79	326 00·5	0 43·8	28 53·0	A	
31	,,	207 17	0 49·0	−28 39·6	B	i	—					
32	Ref. 70, S97	159 04	1 44·5	−29 23·4	A		81	159 04·5	1 44·4	−29 23·6	A	
33	*JHA*, 172	129 04	1 24·6	−19 00·0	A		79	129 13·0	1 24·0	−19 04·8	A	
34	*MLO*, 76	34 15	1 14·0	29 26·4	A		81	34 14·0	1 11·4	29 24·0	A	
35	,,	326 38	0 39·0	29 11·8	A		79	326 38·5	0 37·8	29 10·8	A	
36	,,	211 20	−0 04·0	−28 37·2	B	j	81	211 22·0	−0 07·4	−28 40·6	A	
37	,,	156 12	1 17·0	−29 26·8	A		79	156 02·5	1 15·4	−29 26·0	A	
38	,,	152 30	0 57·0	−28 46·2	A		79	152 26·0	0 55·4	−28 46·8	A	
39	,,	177 26	5 00·6	−28 44·4	A		79	177 26·5	5 00·8	−28 44·0	A	
40	,,	177 48	4 51·7	−28 54·0	A		79	177 47·0	4 53·0	−28 52·8	A	
41	,,	326 09	0 20	28 53·4	A		79	326 07·5	0 19·6	28 52·4	A	
42	,,	304 25	−0 04·0	19 09·6	A		79	304 30·5	−0 04·4	19 12·0	B	p
43	,,	325 12	−0 02	28 46·2	A		—					
44	,	300 50	−0 21·6	18 20·0	C	k	—					

14	15	16	17	18	19	20	21	22	23
° ′	′					′	′	′	
−29 09·6	+0·2	A		M	33·5	0·0	0·0	0·0	B
−18 43·8	0·0	A		T	13·3	±1·0	±0·2	±0·4	E
−18 49·6	+0·4	B		T	13·3	±0·0	±0·2	±0·4	E
−18 58·4	+0·4	B		Tf	13·3	±1·5	{ +0·2 −1·0	+0·6 } −1·0 }	E
−18 49·6	−0·2	A	q	M	6·0	±0·5	±0·2	±0·2	B
−18 18·4	+0·8	B		Ts	6·0	±0·5	±0·2	±0·2	BF
29 25·6	0·0	A	r	X	8·2	{ +3·0 } { −0·5 }	±0·2	{ +0·8 } { −0·2 }	BG
29 25·2	−0·2	A	r	T	8·2	±1·0	{ +0·2 −1·0	+0·4 } −1·0 }	E
28 48·4	0·0	A		M	12·8	±0·5	±0·2	±0·2	B
28 52·8	0·0	A		T	12·8	±1·0	±0·2	±0·4	E
19 08·6	0·0	A		C	2·9	±1·0	±0·6	±0·6	B
−29 08·8	0·0	A		R	79	±2·0	0·0	±0·4	H
−29 26·6	0·0	A		R	88	±2·0	0·0	±0·4	H
−29 13·0	−1·4	A		Z	26·2	{ +0·5 −2·0	+0·2 } −0·0 }	±0·2	J
−28 35·6	−0·2	A		A	16·5	0·0	0·0	0·0	BD
−28 54	0	A		A	0·6	±20	{ +3 } { −4 }	±4	BDK
29 20·4	+3·0	A	s	Tb	11·8	±0·5	±0·2	±0·2	B
19 00·0	0·0	B		S	24·8	±1·5	0·0	±0·6	L
−18 19·2	+1·0	A		M	29·0	0·0	0·0	0·0	B
28 56·8	+0·4	A	t	M	2·0	±1·5	{ +1·0 −2·0	+1·0 } −1·8 }	BD
29 02·4	+0·2	A	t	Q	2·0	±1·5	{ +1·2 −2·0	+1·2 } −1·8 }	BDR
29 19·8	+1·0	A	t	A	2·0	±1·5	{ +1·0 −2·0	+1·0 } −1·8 }	BD
−28 48·4	0·0	B	u	Am	6·3	±2·0	±0·2	±0·6	BDN
29 13·4	0·0	A		A	3·3	±1·0	{ +0·6 −1·0	+0·6 } −1·0 }	BD
29 28·4	0·0	A		A	3·3	±1·0	{ +0·6 −1·0	+0·6 } −1·0 }	BD
29 17·4	+0·2	A		M	5·3	±0·5	±0·4	±0·4	B
−29 27·6	−1·4	A	v	M	6·5	±0·5	±0·2	±0·2	B
29 16·8	−0·4	A		M	2·0	±1·5	±0·8	±0·8	B
29 26·0	0·0	A		M	2·0	±1·5	±0·8	±0·8	B
28 53·0	−1·0	A		M	21·8	0·0	0·0	0·0	B
−28 39·6	0·0	B		A	9·5	±0·5	±0·2	±0·2	BD
−29 23·4	0·0	A		M	18·0	0·0	0·0	0·0	B
−19 04·8	−4·8	A	w	A	19·9	0·0	0·0	0·0	BD
29 25·2	−1·2	A		M	20·3	0·0	0·0	0·0	B
29 11·4	−0·4	A		M	36·5	0·0	0·0	0·0	B
−28 40·6	−3·4	A	x	M	23·1	0·0	0·0	0·0	B
−29 26·0	+0·8	A	y	M	8·5	±0·5	±0·2	±0·2	B
−28 46·8	−0·6	A	y	M	8·5	±0·5	±0·2	±0·2	B
−28 44·2	+0·2	A		M	3·3	±1·0	±0·6	±0·6	B
−28 53·4	+0·6	A		M	3·3	±1·0	±0·6	±0·6	B
28 53·0	−0·4	A		M	3·4	±1·0	±0·6	±0·6	B
19 09·6	0·0	A		M	58·7	0·0	0·0	0·0	B
28 46·2	0·0	A		C	49·1	±0·5	0·0	±0·2	P
18 20·0	0·0	C		A	146	0·0	0·0	0·0	BD

in the comments of column 17, including probable explanations of significant discrepancies between the two surveys.

It is not usually clear whether altitudes quoted by the Thoms have been corrected for the difference in mean terrestrial refraction between the time of day of measurement and that of the hypothesized observation;[98] for example at Mid Clyth a correction *has* been applied in order to bring the observed altitudes into line with those calculated from the Ordnance Survey,[99] whereas in most other cases it appears that the celestial refraction correction appropriate to the time of year and day of assumed use has been applied directly to the measured altitude.[100] Our quoted altitudes are those measured, always between 9h and 17h UT during the summer months, without any correction applied. In Table III we list the differences between our altitude and the Thoms' for those twelve lines where the latter is quoted accurate to 0′·1, both surveys were of status 'A' and the foresight and assumed observing position are unambiguous. These differences are compared with the maximum daylight variation expected from the effects of terrestrial refraction.[101] Four of the lines show significant excesses over the maximum expected amount, by up to 1′·6, although this figure occurs for a line where the Thoms' altitude has decimal zero, and the declination may be in error by up to $\pm 0'\cdot 4$. We conclude that terrestrial refraction variations may well account for a good deal of the altitude discrepancies listed in Table III, but that differences of up to about 1′ still remain in some cases.

Whatever the cause of the altitude discrepancies, it is clear that altitude measurements in general do not appear to be reliable to better than about 1′. We see no reason to suspect our own measurements to be less reliable than those of the Thoms in this respect. Thus we feel that altitudes, and hence declinations, may be subject to a random probable error of up to $\pm 1'$, and that to quote them to 0′·2 displays false accuracy. This is an opinion shared by Patrick.[102]

In the final columns of Table II we consider how greatly the assumed indicated declinations are affected by uncertainties in the observing position (OP), a point which does not seem to have received attention elsewhere. In order to

TABLE III. Differences in altitude measurements by the author and by the Thoms, where both measurements should be reliable (status 'A') and the foresight and assumed observing position are unambiguous.

Line no.	Altitude difference (author − Thoms) ′	Maximum expected altitude difference due to terrestrial refraction effects ′	Numerical excess over maximum expected ′
34	−2·6	1·0	1·6
35	−1·2	1·8	–
28	−0·8	0·1	0·7
33	−0·6	1·0	–
11	−0·2	0·1	0·1
32	−0·1	0·9	–
29	0·0	0·1	–
39	+0·2	0·2	–
1	+0·5	1·6	–
26	+1·0	0·3	0·7
19	+1·3	1·4	–
40	+1·3	0·2	1·1

conform to a fixed code of practice, we have taken the OP as 2 m behind indications such as stone alignments and menhir flat faces, and 2 m behind single menhirs where there is no indication. In the case of alignments, we have allowed for the possibility of the OP being anywhere from 0 to 10 m behind the indication. In all cases, we have assumed a lateral uncertainty of 1 m either way in the OP. We have assumed throughout an observing height of 1·5 m above present ground level, allowing an error of $\pm 0{\cdot}5$ m for differences in observer height and ground level changes. This produces a declination uncertainty of as much as $\pm 0'{\cdot}8$ at Kilmartin (A_1) and Ballinaby, and even more at Corogle Burn. The use of mounds as backsights, as at Brogar, admits four possible OPs in each case: the top at original height, and the front and sides at ground level. Each of these possibilities is used by the Thoms at one line or another; however only in the case of Line 6 is there a *de facto* case (on the grounds of the indication) for choosing any one in preference to any other. Accordingly, we have taken the OPs used by the Thoms but allow for the full range of possibilities (except in the case of Line 6) when estimating the uncertainties in azimuth, altitude and declination.

In column 15 of Table II we list the differences between what are now considered to be the most reliable mean declinations and those that were deduced from the Thoms' measurements alone. These should indicate by how much the declinations used by the Thoms in their analyses of the sightlines should be altered in order to bring them into line with the new, more reliable, values. These corrections, together with the inherent uncertainties in the declination values (column 22), will feature in the discussions that follow in Section 8.

6. *The Foresights and Their Selection*

A convenient basis for the classification and discussion of putative foresights has been laid down by Thom and Thom.[123] It involves separating them into five types: in Type I the lunar limb reappears momentarily in a notch, in Type II it trickles down or up a sloping part of horizon parallel to its path, in Type III it runs into or emerges from a sharp corner, in Type IV it grazes the rounded shoulder of a hill, and in Type V it appears or disappears behind a small irregularity in an otherwise relatively flat part of horizon. If, in place of a lunar limb we think in terms of any given line of constant declination, then the classification extends to any horizon feature. It is simply a function of the feature's shape and the slope of the declination lines behind it, and is independent of any particular astronomical interpretation. We follow this classification system here, but (unlike the Thoms) make no distinction between upper and lower limb phenomena, since this brings in an assumed function for any particular horizon feature.

In the case of Line 16 (Corogle Burn) there is in fact no horizon foresight, the horizon being nearby, flat and featureless. The horizon point considered by the Thoms is defined only by the indication on the ground, which explains the large uncertainties listed in columns 20–22 of Table II. This line should not rightly be included in the present analysis since it does not provide data with which to test the overall Level 4 hypothesis identified in Section 2.[124] We shall not reconsider a further six foresights which we were unable to see and survey and where we can add nothing to the information provided by the Thoms.

The sites concerned are Mid Clyth, Lundin Links, Escart, Blakeley Moss and Parc-y-Meirw. In the case of the last site we would draw attention to the very great distance of the proposed foresight (146 km) and to consequent doubts about its plausibility.[125]

We have reassessed the classification of the remaining thirty-seven foresights on the basis of our own surveys and those of the Thoms. To do this lines of constant declination were superimposed upon horizon plots in which the gaps between surveyed points had been filled in using enlargements of 200 mm and 400 mm photographs taken on site. The results are listed in column 5 of Table IV. In some cases a degree of subjective judgement enters, as for example in deciding when a "small irregularity" becomes sufficiently large that a classification other than Type V is appropriate. In such cases we have attempted to follow the Thoms' own criteria as judged from entries in their list of foresights at various places.[126] Occasionally where a foresight is not included in this list, our classification is different from the one that would appear to be correct from inspection of the Thoms' profile diagram for that line. Examples are A_2 at High Park (Type I, not Type II) and A at Dunadd (Type III, not Type II). We attribute such discrepancies to oversimplification in the Thoms' horizon diagrams between surveyed points, an effect which is

TABLE IV. Foresights proposed by the Thoms, their classification, and the number and classification of all equally plausible horizon features within the same lunar bands.

Column headings

1 Sightline number
2 Overall status (see Section 4 and Table I)
3 Foresight reference used by the Thoms
4 Reference to the Thoms' profile diagram
5 Foresight type (see Section 6)
6 Azimuth range of indication
7 Azimuth range of lunar band
8 Total number of features of each type in the indicated azimuth range (IAR) (see Section 6). * indicates that this includes the Thoms' foresight
9 Total number of features of each type in the adjacent azimuth range (AAR) (see Section 6). * as above
10 Total number of features of each type which are not indicated. * as above
11 Overall total number of features within lunar band
12 Comments

Key to column 12 (comments)

a Only the azimuth range 190°–194° was resurveyed and photographed. The quoted width of the lunar band is approximate and the quoted number of features is a minimum estimate.

b The Thoms' feature does not show up on our 200 mm photograph and, but for their having included it, we would not have done. Thus other features of prominence equal to that of the proposed foresight might have passed unnoticed.

c The profile used is in fact obscured from behind the indication by local ground, although it can be seen from a few metres to the west (see Section 3).

d At azimuths greater than 163° the distant profile (as viewed from behind the indication) is obscured by local ground. We have thus taken this part of the horizon to be featureless.

e Only the azimuth range 176°–187° was resurveyed and photographed. The quoted width of the lunar band is a minimum estimate, and the band certainly extends for some degrees to either side of this. The actual number of features within the band is also in excess of that quoted.

TABLE IV.

1	2	3	4	5	6	7	8						9						10						11	12
					° °	° °	I	II	III	IV	V	Total	I	II	III	IV	V	Total	I	II	III	IV	V	Total		
1	Y	—	—	I		188–195													1*	1	0	1	0	(3)	3	a
2	WA	Hellia	*MRBB*, 124	I	229–231	226–230	0	0	0	0	0	(0)	5*	1	1	0	0	(7)							7	
3	XA	Hellia	*MRBB*, 124	I		226–230													5*	1	1	0	0	(7)	7	
4	XA	Hellia	*MRBB*, 124	I		226–230													5*	1	1	0	0	(7)	7	
5	WA	Mid Hill	*MRBB*, 124	V	133–135	132–137	0	0	0	0	0	(0)	0	0	0	0	1*	(1)							1	
6	A	Mid Hill	*MRBB*, 124	V	133–135	132–137	0	0	0	0	1*	(1)	0	0	0	0	0	(0)							1	
7	WA	Kame U	*MRBB*, 124	II	27– 31	22– 28	0	0	0	0	0	(0)	0	2*	0	0	0	(2)							2	
8	WC	Kame L	*MRBB*, 124	II	26– 30	22– 28	0	0	0	0	0	(0)	0	2*	0	0	0	(2)							2	
9	WC	Ravie	*MRBB*, 125	V	337–339	335–339	0	0	0	0	0	(0)	0	0	0	0	1*	(1)							1	b
10	B	Ravie	*MRBB*, 125	V	336–338	335–339	0	0	0	0	1*	(1)	0	0	0	0	0	(0)							1	b
11	Z	—	*MRBB*, 129	II	307–308	305–308	0	1*	1	0	0	(2)	0	0	1	0	0	(1)							3	
14	Z	A_1	*MLO*, 69	IV	188–192	188–197	4	0	0	2*	0	(6)	2	0	1	1	0	(4)							10	c
15	WB	A_1	*MLO*, 69	II	163–165	160–171	0	0	0	0	0	(0)	1	1*	0	0	0	(2)							2	d
17	A	A	*MRBB*, 174	III	29– 32	30– 34	0	0	1*	0	1	(2)	0	0	0	0	1	(1)							3	
19	A	A_1	*MLO*, 39	V	230–236	231–234	0	6	1	0	3*	(10)													10	b
20	WA	A_1	*MLO*, 46	II	314–316	315–318	0	1	1	0	0	(2)	0	2*	1	0	0	(3)							5	
21	A	A_1	*MLO*, 46	II	316–318	315–318	0	2*	1	0	0	(3)	0	1	1	0	0	(2)							5	
22	WA	A_1	*MLO*, 46	II	321–324	315–318							0	3*	2	0	0	(5)							5	
23	WA	A_2	*MLO*, 46	I	202–207	203–208	2	0	0	2	0	(4)	2*	0	0	1	0	(3)							7	
24	A	A_1	*MLO*, 52	II }	321–323	319–323	0	1*	0	1*	0	(2)	1	0	0	0	0	(1)							3	
25	A	A_2	*MLO*, 52	IV }																						
26	WA	A	*MLO*, 64	III	327–329	322–326							0	2	2*	0	0	(4)							4	
27	WA	A	*MLO*, 65	I	205–206	203–208	0	1	0	0	0	(1)	2*	0	0	1	0	(3)							4	
28	A	B	*MRBB*, 170	I }	324–330	325–330	1*	3*	3	1	1	(9)													9	
29	A	C	*MRBB*, 170	II }																						
30	B	A	*MLO*, 66	II	325–326	325–329	0	1*	0	0	0	(1)	1	4	1	0	0	(6)							7	
32	Z	—	Ref. 70, S97	I		157–162													7*	3	0	1	0	(11)	11	
33	C	R	*JHA*, 172	III	128–129	127–130	0	0	1*	0	0	(1)	0	0	0	0	0	(0)							1	
34	Y	A_2	*MLO*, 62	II		33– 36													1	1*	0	0	0	(2)	2	
35	Y	A	*MLO*, 61	II		325–329													1	2*	0	1	0	(4)	4	
36	WC	A_2	*MLO*, 57	I	212–213	207–212							1*	1	0	0	0	(2)							2	
37	Y	A_1	*MLO*, 60	II }		152–157													1*	4*	1	0	1	(7)	7	
38	Y	A_2	*MLO*, 60	I }																						
39	Y	A_1	*MLO*, 62	IV }		176–187													5*	2	1	3*	3	(14)	14	e
40	Y	A_2	*MLO*, 62	I }																						
41	WA	B	*MLO*, 63	I	{ 319–321 / 331–333 }	324–328							1*	0	3	0	0	(4)							4	
42	WA	A_1	*MLO*, 72	II	303–304	303–305	0	0	6	0	0	(6)	0	1*	0	0	0	(1)							7	

FIG. 5. 400 mm photograph of Hellia as viewed from the centre of the ring at Brogar. Feature numbers are referred to in the text; Feature 4 is the Thoms' proposed foresight. The line represents a line of constant declination.

obvious from several of our profile photographs and which can be seen in one case by comparing the 400 mm photograph of Hellia from Brogar (Figure 5) and the Thoms' diagram of the foresight.[127]

Each of the thirty-seven given foresights is contained within one of the "lunar bands"[128] as viewed from a particular observing position. There are four cases where two given foresights occur in the same band, and so we are concerned with a total of thirty-three bands, as viewed from particular observing positions. In order to tackle question (3) of Section 1 we wish to investigate all other classifiable horizon features within these same bands, and to take indications into account. Thus we define the indicated azimuth range (IAR) where an indication exists, and take an adjacent azimuth range (AAR) extending for 5° in azimuth in each direction beyond the limits of the IAR. In Table IV we identify for each of the thirty-three bands the IAR (where appropriate), the azimuth limits of the band itself, and the number of horizon features of each type occurring within the IAR, within the AAR, and outside both (*i.e.* non-indicated features). In identifying horizon features for inclusion we have been guided by those actually included by the Thoms as foresights. A degree of subjectivity still enters, especially in deciding how small a feature should be before it is ignored. In each case we have only included features of at least roughly equal prominence to the Thoms' foresight(s) in the band concerned. Even this procedure presented problems with Ravie Hill at Brogar and at Kintraw, where the Thoms' foresights were so small that they did not even show up on our photographs, and we would not have surveyed them but for their use by the Thoms. Thus in these

FIG. 6. 400 mm photograph of the horizon at Kintraw. Feature numbers are referred to in Fig. 7 and in Table V; Feature 9 is the Thoms' A_1 and Feature 7 the Thoms' A_2.[129] The lines represent lines of constant declination, but not the edges of the lunar band.

cases features of equal prominence to that of the proposed foresight might have passed unnoticed.

In some cases, such as Mid Hill at Brogar and Dunskeig, the feature chosen by the Thoms is the only admissible one in the entire lunar band, and in others, such as Wormadale Hill and Beinn an Tuirc, it is undoubtedly the most convincing. However, in many other cases features of prominence similar to or much greater than the chosen foresight have apparently been ignored by the Thoms. In Figure 5 we show a 400 mm photograph of Hellia as seen from the centre of the ring at Brogar. Feature 4 is the foresight used by the Thoms, and it is classified as Type I, although it is the bottom of a rounded dip almost imperceptible to the naked eye. We have marked on the photograph the six other features we regard as equally plausible and have included in Table IV. The sloping line represents a line of constant declination. We have classified Features 1, 2, 4, 5 and 7 as Type I in accordance with the designation of Feature 4 as such; Feature 3 is Type II and Feature 6 is Type III. (Note that Feature 7, the junction with nearer ground, varies from place to place at Brogar, and from each of the three observing positions will be somewhat differently placed from its position in the photograph.) A typical analysis, that at Kintraw, is presented in full in Figures 6 and 7 and Table V.

Declinations have been calculated for all the horizon features listed in Table IV. In most cases they have been surveyed directly, and declinations should be reliable to 1′, but exceptionally values had to be extrapolated from those of adjacent surveyed points using the photographs, and in these cases larger errors are possible. Values obtained are listed in Section 8, where they are analysed. The question of the limits of each lunar band, beyond which features were not included in Table IV, is also discussed in Section 8.

7. *The Analysis of Lunar Declinations*

In this section we consider in general terms the analysis of measured declinations within the lunar bands, reformulating the relevant expressions in a way that will facilitate the statistical discussion of the following section. It may also help to ease confusion between the differing notations of Morrison[130] and Thom and Thom.[131]

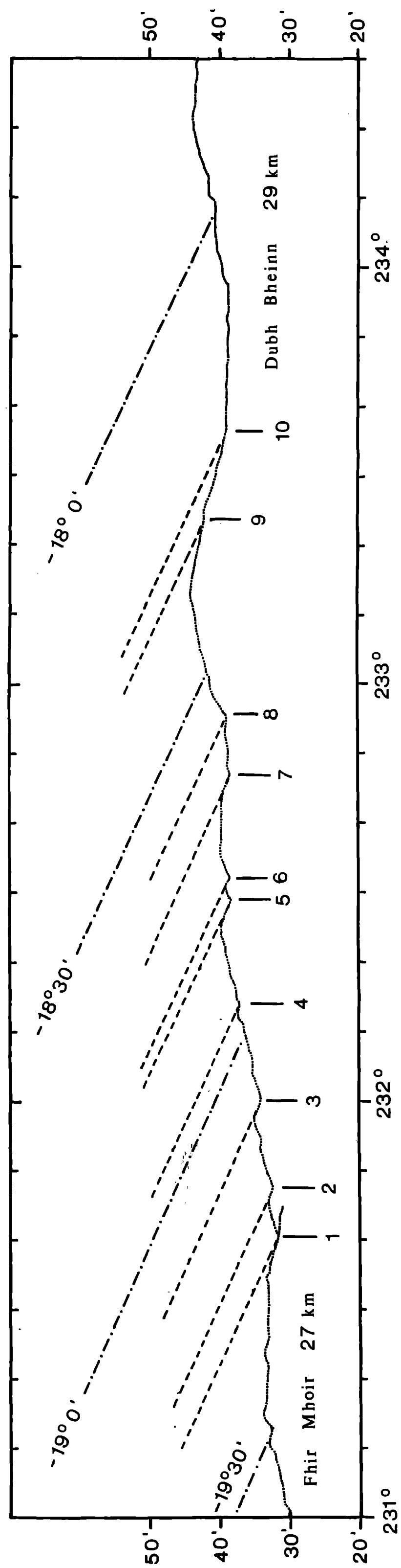

FIG. 7. Plot of the horizon at Kintraw obtained using an enlargement of the photograph in Fig. 6. Features 1, 2, 3, 5, 6 and 7 were classified as Type II, Feature 8 as Type III and Features 4, 9 and 10 as Type V.

TABLE V. Analysis of horizon features within the lunar band which includes foresight A_1 at Kintraw.

Column headings

1 Feature identifier (compare Figures 6 and 7)
2 Type
3 Azimuth
4 Altitude
5 Lunar declination deduced from these
6 Difference β from mean standstill (see Sections 7 and 8)
7 Comments

1	2	3	4	5	6	7
		° ′	° ′	° ′	′	
1	II	231 40·5	0 32·0	−19 17·8	−31·0	
2	II	231 47·5	0 33·0	−19 13·4	−26·6	
3	II	232 01·0	0 35·0	−19 05·2	−18·4	
4	V	232 14·0	0 37·8	−18 56·2	− 9·4	
5	II	232 29·0	0 39·2	−18 48·0	− 1·2	
6	II	232 32·5	0 39·0	−18 46·6	+ 0·2	
7	II	232 46·5	0 39·4	−18 39·6	+ 7·2	Thom's A_2
8	III	232 56·0	0 40·0	−18 34·6	+12·2	
9	V	233 25·0	0 42·8	−18 18·2	+28·6	Thom's A_1
10	V	233 34·5	0 40·4	−18 16·0	+30·8	

From the measured altitude h_0 we calculate a geocentric altitude h using

$$h = h_0 - R_0 + p_0,$$

where the refraction correction R_0 (h_0) corresponds[89] to a standard assumption of temperature 10°C and pressure 1005 mb, and the parallax correction $p_0 = 56'{\cdot}9 \cos (h_0 - R_0)$ corresponds to a standard horizontal parallax of 56′·9. We then proceed to calculate a geocentric declination δ_0 from the measured azimuth A, geocentric altitude h and site latitude ϕ using the standard formula

$$\sin \delta_0 = \sin \phi \sin h + \cos \phi \cos h \cos A.$$

From δ_0 we derive $\beta = \delta_0 - \delta_m$, the difference of δ_0 from the relevant mean standstill declination for a provisional epoch of 2000 B.C. The four values of δ_m[132] are

$$\pm(\epsilon_0 + i) = (23° \; 55'{\cdot}65 + 0'{\cdot}15 + 5° \; 8'{\cdot}7) = \pm(29° \; 4'{\cdot}5) \text{ and}$$
$$\pm(\epsilon_0 - i) = (23° \; 55'{\cdot}65 - 0'{\cdot}15 - 5° \; 8'{\cdot}7) = \pm(18° \; 46'{\cdot}8).$$

In this manner we eliminate from our data set of β values any implicit (and variable) assumptions about refraction and parallax which are dependent upon the event assumed to have been observed, as well as any hidden predilections about the approximate date of use of the sightlines.

For any particular lunar event that might have been recorded using a horizon foresight, we can calculate the expected or "target" value of β, which, following the Thoms, we call Q. It will be given by the sum of terms of the following form, each representing the mean value of a particular quantity when averaged for that event over a large number of standstills.

(i) "Δ" term (mean lunar perturbation). Numerically equal to 8′·6 at equinoxes and 10′·0 at solstices, sign that of the Thoms' "Δ".[133]

(ii) "s" term (mean semidiameter). Numerically equal to 15′·4 at equinoxes and 15′·6 at solstices, sign that of the Thoms' "s".[134]

TABLE VI. Expected or "target" values Q of the difference β of measured declinations from the mean standstill values for each of the nominal lunar events considered by the Thoms.

Nominal declination	Rise Month	Rise Hour	Rise Temp.	Set Month	Set Hour	Set Temp.	Value of terms (i)	(ii)	(iii)	(viii)	Total value of Q					Grade (see text)
			°C			°C	arc minutes				arc minutes					
$+(\varepsilon+i+\Delta+s)$	Sept.	20h	10	Mar.	4h	5	+ 8·6	+15·4	+0·5	+0·2	+24·7	$+G$	$+\varepsilon_T$	$-3{\cdot}9\xi$	$-0{\cdot}7\eta$	(**)
$+(\varepsilon+i+\Delta\ \ )$							+ 8·6		+0·5	+0·2	+ 9·3	$+G$	$+\varepsilon_T$	$-3{\cdot}9\xi$	$-0{\cdot}7\eta$	(†)
$+(\varepsilon+i+\Delta-s)$	Sept.	20h	10	Mar.	4h	5	+ 8·6	−15·4	+0·5	+0·2	− 6·1	$+G$	$+\varepsilon_T$	$-3{\cdot}9\xi$	$-0{\cdot}7\eta$	(**)
$+(\varepsilon+i\ \ +s)$							− 0·7	+15·5		+0·3	+15·1	$+G$	$+\varepsilon_T$	$-3{\cdot}9\xi$	$-0{\cdot}3\eta$	(†)
$+(\varepsilon+i\ \ \ \ )$							− 0·7			+0·3	− 0·4	$+G$	$+\varepsilon_T$	$-3{\cdot}9\xi$	$-0{\cdot}3\eta$	(††)
$+(\varepsilon+i\ \ -s)$							− 0·7	−15·5		+0·3	−15·9	$+G$	$+\varepsilon_T$	$-3{\cdot}9\xi$	$-0{\cdot}3\eta$	(†)
$+(\varepsilon+i-\Delta+s)$	Dec.	14h	5	Dec.	10h	5	−10·0	+15·6	−0·5	+0·4	+ 5·5	$+G$	$+\varepsilon_T$	$-3{\cdot}9\xi$	$+0{\cdot}1\eta$	
$+(\varepsilon+i-\Delta\ \ )$							−10·0		−0·5	+0·4	−10·1	$+G$	$+\varepsilon_T$	$-3{\cdot}9\xi$	$+0{\cdot}1\eta$	(†)
$+(\varepsilon+i-\Delta-s)$	Dec.	14h	5	Dec.	10h	5	−10·0	−15·6	−0·5	+0·4	−25·7	$+G$	$+\varepsilon_T$	$-3{\cdot}9\xi$	$+0{\cdot}1\eta$	
$+(\varepsilon-i+\Delta+s)$	Dec.	16h	5	Dec.	8h	5	+10·0	+15·6	−0·5	+0·4	+25·5	$+G$	$+\varepsilon_T$	$-2{\cdot}5\xi$	$-0{\cdot}1\eta$	
$+(\varepsilon-i+\Delta\ \ )$							+10·0		−0·5	+0·4	+ 9·9	$+G$	$+\varepsilon_T$	$-2{\cdot}5\xi$	$-0{\cdot}1\eta$	(†)
$+(\varepsilon-i+\Delta-s)$	Dec.	16h	5	Dec.	8h	5	+10·0	−15·6	−0·5	+0·4	− 5·7	$+G$	$+\varepsilon_T$	$-2{\cdot}5\xi$	$-0{\cdot}1\eta$	
$+(\varepsilon-i\ \ +s)$							+ 0·7	+15·5		+0·3	+16·5	$+G$	$+\varepsilon_T$	$-2{\cdot}5\xi$	$+0{\cdot}3\eta$	(†)
$+(\varepsilon-i\ \ \ \ )$							+ 0·7			+0·3	+ 1·0	$+G$	$+\varepsilon_T$	$-2{\cdot}5\xi$	$+0{\cdot}3\eta$	(††)
$+(\varepsilon-i\ \ -s)$							+ 0·7	−15·5		+0·3	−14·5	$+G$	$+\varepsilon_T$	$-2{\cdot}5\xi$	$+0{\cdot}3\eta$	(†)
$+(\varepsilon-i-\Delta+s)$	Sept.	22h	10	Mar.	2h	5	− 8·6	+15·4	+0·5	+0·2	+ 7·5	$+G$	$+\varepsilon_T$	$-2{\cdot}5\xi$	$+0{\cdot}7\eta$	(*)
$+(\varepsilon-i-\Delta\ \ )$							− 8·6		+0·5	+0·2	− 7·9	$+G$	$+\varepsilon_T$	$-2{\cdot}5\xi$	$+0{\cdot}7\eta$	(†)
$+(\varepsilon-i-\Delta-s)$	Sept.	22h	10	Mar.	2h	5	− 8·6	−15·4	+0·5	+0·2	−23·3	$+G$	$+\varepsilon_T$	$-2{\cdot}5\xi$	$+0{\cdot}7\eta$	(*)
$-(\varepsilon-i-\Delta-s)$	Mar.	3h	5	Sept.	21h	10	+ 8·6	+15·4	+0·5	+0·2	+24·7	$+G$	$-\varepsilon_T$	$+2{\cdot}5\xi$	$-0{\cdot}7\eta$	(*)
$-(\varepsilon-i-\Delta\ \ )$							+ 8·6		+0·5	+0·2	+ 9·3	$+G$	$-\varepsilon_T$	$+2{\cdot}5\xi$	$-0{\cdot}7\eta$	(†)
$-(\varepsilon-i-\Delta+s)$	Mar.	3h	5	Sept.	21h	10	+ 8·6	−15·4	+0·5	+0·2	− 6·1	$+G$	$-\varepsilon_T$	$+2{\cdot}5\xi$	$-0{\cdot}7\eta$	(*)
$-(\varepsilon-i\ \ -s)$							− 0·7	+15·5			+14·8	$+G$	$-\varepsilon_T$	$+2{\cdot}5\xi$	$-0{\cdot}3\eta$	(†)
$-(\varepsilon-i\ \ \ \ )$							− 0·7				− 0·7	$+G$	$-\varepsilon_T$	$+2{\cdot}5\xi$	$-0{\cdot}3\eta$	(††)
$-(\varepsilon-i\ \ +s)$							− 0·7	−15·5			−16·2	$+G$	$-\varepsilon_T$	$+2{\cdot}5\xi$	$-0{\cdot}3\eta$	(†)
$-(\varepsilon-i+\Delta-s)$	June	21h	15	June	3h	10	−10·0	+15·6	−0·5	−0·2	+ 4·9	$+G$	$-\varepsilon_T$	$+2{\cdot}5\xi$	$+0{\cdot}1\eta$	
$-(\varepsilon-i+\Delta\ \ )$							−10·0		−0·5	−0·2	−10·7	$+G$	$-\varepsilon_T$	$+2{\cdot}5\xi$	$+0{\cdot}1\eta$	(†)
$-(\varepsilon-i+\Delta+s)$	June	21h	15	June	3h	10	−10·0	−15·6	−0·5	−0·2	−26·3	$+G$	$-\varepsilon_T$	$+2{\cdot}5\xi$	$+0{\cdot}1\eta$	
$-(\varepsilon+i-\Delta-s)$	June	22h	15	June	2h	10	+10·0	+15·6	−0·5	−0·2	+24·9	$+G$	$-\varepsilon_T$	$+3{\cdot}9\xi$	$-0{\cdot}1\eta$	
$-(\varepsilon+i-\Delta\ \ )$							+10·0		−0·5	−0·2	+ 9·3	$+G$	$-\varepsilon_T$	$+3{\cdot}9\xi$	$-0{\cdot}1\eta$	(†)
$-(\varepsilon+i-\Delta+s)$	June	22h	15	June	2h	10	+10·0	−15·6	−0·5	−0·2	− 6·3	$+G$	$-\varepsilon_T$	$+3{\cdot}9\xi$	$-0{\cdot}1\eta$	
$-(\varepsilon+i\ \ -s)$							+ 0·7	+15·5			+16·2	$+G$	$-\varepsilon_T$	$+3{\cdot}9\xi$	$+0{\cdot}3\eta$	(†)
$-(\varepsilon+i\ \ \ \ )$							+ 0·7				+ 0·7	$+G$	$-\varepsilon_T$	$+3{\cdot}9\xi$	$+0{\cdot}3\eta$	(††)
$-(\varepsilon+i\ \ +s)$							+ 0·7	−15·5			−14·8	$+G$	$-\varepsilon_T$	$+3{\cdot}9\xi$	$+0{\cdot}3\eta$	(†)
$-(\varepsilon+i+\Delta-s)$	Mar.	4h	5	Sept.	20h	10	− 8·6	+15·4	+0·5	+0·2	+ 7·5	$+G$	$-\varepsilon_T$	$+3{\cdot}9\xi$	$+0{\cdot}7\eta$	(**)
$-(\varepsilon+i+\Delta\ \ )$							− 8·6		+0·5	+0·2	− 7·9	$+G$	$-\varepsilon_T$	$+3{\cdot}9\xi$	$+0{\cdot}7\eta$	(†)
$-(\varepsilon+i+\Delta+s)$	Mar.	4h	5	Sept.	20h	10	− 8·6	−15·4	+0·5	+0·2	−23·3	$+G$	$-\varepsilon_T$	$+3{\cdot}9\xi$	$+0{\cdot}7\eta$	(**)

(iii) Mean parallax correction. $+0'{\cdot}5 \cos h\ \partial\delta/\partial h$ at equinoxes (for which mean horizontal parallax is $56'{\cdot}4$ rather than $56'{\cdot}9$) and $-0'{\cdot}5 \cos h\ \partial\delta/\partial h$ at solstices (for which mean horizontal parallax is $57'{\cdot}4$).[135]

(iv) Correction for mean graze effect. $+G\partial\delta/\partial h$ where G is an unknown factor whose value, as deduced by the Thoms, is typically in the range $0'{\cdot}0$ to $1'{\cdot}0$.[136]

(v) Year correction. Numerically equal to $\epsilon_T = \epsilon - \epsilon_0 = -0'{\cdot}64\,T - 0'{\cdot}004\,T^2$ where T is the date of use measured in centuries forward from 2000 B.C.[137] Correction is $+\epsilon_T$ for northern and $-\epsilon_T$ for southern declinations.

(vi) Mean extrapolation correction. Numerically equal to $3'{\cdot}9\xi$ at major standstill and $2'{\cdot}5\xi$ at minor standstill,[138] where $\xi = 0$ if the decrement was eliminated by extrapolation;[139] $\xi = 1$ if not. Sign negative for northern and positive for southern declinations.

(vii) Mean corrections for standstill not coinciding exactly with equinox/solstice or monthly extreme. Sum of two terms $c_1\eta$ (with c_1 numerically equal to $0'{\cdot}3$, sign opposite to that of "i") and $c_2\eta$ (with c_2 numerically equal to $0'{\cdot}4$, sign opposite to that of the Thoms' "Δ"),[140] where $\eta = 0$ if the decrements were eliminated by repeated observation over several standstills, $\eta = 1$ if not.

(viii) Correction for mean atmospheric conditions at the time of observation. This is discussed below.

The signs of the corrections under (iii) and (iv) merit some explanation. For an event for which the mean refraction and parallax corrections are respectively R and p, rather than R_0 and p_0, the appropriate geocentric altitude is given by $h_0 - R + p$, exceeding h by $(-R + R_0 + p - p_0)$; thus the appropriate geocentric declination δ in fact exceeds δ_0 by $(-R + R_0 + p - p_0)\ \partial\delta/\partial h$. Since we demand to know the expected value of $\delta_0 - \delta_m$ rather than that of $\delta - \delta_m$, the target values Q of $\delta_0 - \delta_m$ must be *decreased* by $(-R + R_0 + p - p_0)\ \partial\delta/\partial h$, *i.e.* increased by $(R - R_0 - p + p_0)\ \partial\delta/\partial h$, in order to take account of this effect. Thus for example at equinoxes, where $p - p_0 = -0'{\cdot}5 \cos h$, the correction to Q is $+0'{\cdot}5 \cos h\ \partial\delta/\partial h$. The graze effect increases the refraction coefficient, so that for graze G we have $R - R_0 = G$ and the correction to Q is $+G\ \partial\delta/\partial h$.

We can replace the site-dependent terms in (i)–(viii) by their average values in order to simplify the subsequent analysis, at the risk of introducing certain effectively random errors. Values of $\partial\delta/\partial h$ range from $0{\cdot}85$ to $1{\cdot}0$, so that taking it to be $0{\cdot}925$ will introduce an error of at most $\pm 0'{\cdot}04$ into term (iii) and about $\pm 0'{\cdot}1$ into term (iv). To simplify what follows we redefine G to have $0{\cdot}925$ times its old ("Thom") value. Horizon altitude h is never above about 5°, so the error introduced by taking $\cos h$ to be unity in term (iii) is utterly negligible. There is a hidden latitude dependence in the expression for mean horizontal parallax itself,[141] but here again errors are negligible.

For each lunar event, observed at moonrise or moonset, there is only one approximate time of year and day when the observation could have been made. (There is in each case an alternative theoretical possibility, but for equinoctial observations this would involve observing in broad daylight and for solstitial observations this would involve observing the new Moon.[142]) The appropriate times are listed in columns 2 and 3 of Table VI; the quoted hour of day may be inaccurate by up to about ±2 h, depending on foresight azimuth and site latitude, but these variations are negligible for mean refraction estimates. We assume mean conditions to be a temperature of 5°C around 3 h in March and 9 h and 15 h in December; 10°C around 3 h in June and 21 h in September; 15°C around 21 h in June, and a pressure of 1005 mb throughout.[143] Decreasing the temperature decreases the geocentric altitude, and hence the appropriate geocentric declination δ; thus we expect δ_0 to be too large and we must increase the target value Q. The size of the altitude correction is about 0′·7 when $h = 0°$, 0′·5 when $h = 1°$ and 0′·2 when $h = 5°$.[89] Thus we adjust the target declination values by +0′·4 for observations at 5°C and −0′·4 for those at 15°C, a procedure which will introduce random errors of up to about ±0′·3, with a root mean square error of about 0′·2.[144] As a final simplifying assumption, for those cases listed in Table VI where the rising and setting lines would require corrections differing by 0′·4, we have adopted a mean value in order to avoid separating the two cases. This will introduce a further r.m.s. error of about 0′·1 overall, and the various approximations discussed above will introduce a total possible error of up to about ±0′·6, with an r.m.s. of at most 0′·3. At this small cost, we have confined all the site dependence to the measured declination differences β, and all dependence upon the postulated function of any sightline to the expected or target values Q.

In Table VI we list the values of Q for each of the lunar events considered by the Thoms, using the approximations detailed above. These represent the maximal set of target values for β to be considered in any statistical discussion. Practical considerations lead us to grade these targets in terms of the likelihood of their actually having been recorded, as follows:

(**) [most likely] the most extreme northerly and southerly declinations attained by the Moon, observed on either limb (2 targets, major standstill bands only);

(*) the other most extreme declinations attained at the equinoxes, observed on either limb (2 targets, minor standstill bands only);

( ) all other events capable of direct observation (2 targets in each band);

(†) events which could only be recorded by averaging observations of two other events (*i.e.* averaging between equinoctial and solstitial observations in an attempt to eliminate the lunar perturbation, or averaging between upper and lower limb observations to obtain the exact centre of the lunar disc) (4 targets in each band);

(††) [least likely] events requiring both of the above averaging processes, that is, requiring observations of four directly observable events before they can be recorded (1 target in each band).

This suggests five distinct hypotheses that may be tested:

(A) any of the thirty-six lunar events may have been recorded;

(B) any of the thirty-two lunar events excluding (††) may have been recorded;

(C) any of the sixteen directly observable events may have been recorded (*i.e.* (†) and (††) are excluded);

(D) any of the equinoctial extreme declinations may have been recorded, using either limb (*i.e.* (*) and (**) only);

(E) either the most extreme northerly or southerly declination may have been recorded, using either limb (*i.e.* (**) only).

Even if a foresight *was* used to record precisely the particular lunar event for which we now test, and we can remeasure its declination precisely from the exact observing position used by the constructors, the value of β we measure will still deviate from the expected value Q, by an amount depending upon the extent to which each of the terms (i)–(viii) deviated from its mean value on that particular occasion when the sightline was set up (or the mean of these deviations if observations of the same event over several standstills were first averaged by the constructors). The extent and nature of the fluctuations in each term are as follows:

(i) By up to $\pm 0'{\cdot}7$, with r.m.s. variation about $0'{\cdot}4$. Effectively random.[145]

(ii) By up to $\pm 0'{\cdot}7$ at equinoxes, with r.m.s. $0'{\cdot}5$; by up to $\pm 1'{\cdot}1$ at solstices, with r.m.s. $0'{\cdot}8$. Sinusoidal variation with period 179 years.[146]

(iii) By up to $\pm 2'{\cdot}5$ at equinoxes, with r.m.s. $1'{\cdot}6$; by up to $\pm 3'{\cdot}6$ at solstices, with r.m.s. $2'{\cdot}4$. Effectively sinusoidal with period 179 years.[147]

(iv) Unknown.

(v) Depends upon the spread of dates of construction of the sightlines investigated. For example a 200-year spread gives a variation of up to about $\pm 0'{\cdot}6$, with r.m.s. $0'{\cdot}3$; a 500-year spread gives up to about $\pm 1'{\cdot}6$, with r.m.s. $0'{\cdot}9$; and a 1000-year spread gives up to about $\pm 3'{\cdot}2$, with r.m.s. $1'{\cdot}8$.[148] Effectively random (in the absence of independent dating evidence).

(vi) By up to the value of the decrement in the opposite sense, and somewhat more in the same sense. R.m.s. variations about $2'{\cdot}8$ for major standstill and $1'{\cdot}8$ for minor standstill. Effectively random; case $\xi = 1$ only. (Even in the case $\xi = 0$, the extrapolation process could introduce errors caused by variations in the extrapolation length; however, we disregard this possibility.)

(vii) By up to the values of the decrements in the opposite sense, and somewhat more in the same sense. Overall r.m.s. variation about $0'{\cdot}4$. Effectively random; case $\eta = 1$ only.

(viii) By up to about $\pm 1'{\cdot}2$, with r.m.s. variation about $0'{\cdot}7$.[149] Truly random.

By concatenating the variations in the individual terms, we can arrive at estimates of the variation in each Q under each of the following hypotheses:

(1) Sightlines were set up after observations of a single occurrence of the event in question, without the use of extrapolation techniques ($\xi = 1$; $\eta = 1$).

(2) Sightlines were set up after observations of a single occurrence of the event in question, using extrapolation to determine the theoretical extreme ($\xi = 0$; $\eta = 1$).

(3) Sightlines were set up only after comparing the observations of occurrences of the event over a complete 179-year cycle, in order to eliminate the periodic variations ($\xi = 0$; $\eta = 1$).

(4) As (3), but from the observations the decrements c_1 and c_2 were also eliminated ($\xi = 0$; $\eta = 0$).

TABLE VII. Estimates of random variations in Q under various hypotheses. Possible (but unknown) random variation in the graze effect has been disregarded.

	200-year span Maximum variation		500-year span Maximum variation		1000-year span Maximum variation	
	′	r.m.s. ′	′	r.m.s. ′	′	r.m.s. ′
Hypothesis (1):						
$\pm(\epsilon+i+\Delta)+s$	±10	3·7	±11	3·8	±13	4·1
$\pm(\epsilon+i+\Delta)-s$	± 9	3·2	±10	3·3	±12	3·7
$\pm(\epsilon+i-\Delta)+s$	±12	4·4	±13	4·5	±14	4·8
$\pm(\epsilon+i-\Delta)-s$	±10	3·4	±11	3·5	±12	3·8
$\pm(\epsilon-i+\Delta)+s$	±10	3·9	±11	4·0	±13	4·3
$\pm(\epsilon-i+\Delta)-s$	± 8	2·7	± 9	2·8	±11	3·2
$\pm(\epsilon-i-\Delta)+s$	± 9	3·0	±10	3·1	±12	3·5
$\pm(\epsilon-i-\Delta)-s$	± 8	2·4	± 9	2·5	±10	3·0
Hypothesis (2):						
$\pm(\epsilon\pm(i+\Delta))+s$	± 6·5	2·4	± 7·5	2·6	± 9·0	3·0
$\pm(\epsilon\pm(i+\Delta))-s$	± 5·0	1·5	± 6·0	1·7	± 7·5	2·3
$\pm(\epsilon\pm(i-\Delta))+s$	± 8·0	3·4	± 9·0	3·5	±10·5	3·9
$\pm(\epsilon\pm(i-\Delta))-s$	± 5·5	1·9	± 6·5	2·1	± 8·5	2·6
Hypothesis (3):						
All directly observable events	± 4·0	0·6	± 5·0	1·0	± 6·5	1·9

Under each hypothesis, the errors are effectively random and thus the uncertainties in each Q represent unavoidable random noise in the data, which is further compounded by any lack of precision on the constructors' part and any uncertainties in our remeasurement (see Section 5).

In Table VII we list values of the random variations in Q for directly observable events under each of Hypotheses (1)–(3). (For (†) and (††) events we must compound the r.m.s. variations in the directly observable events which are averaged in order to record them.) The values quoted are minimum estimates, since they disregard the unknown variation in term (iv). Under Hypothesis (1) all the other terms are compounded;[150] under Hypothesis (2) term (vi) is omitted. In the case of Hypothesis (3) we note that over one cycle a maximum of five observations of the same event on the same foresight is possible, even under perfect conditions.[151] The observers would have experienced a background random variation in declination of (at least) up to $\pm 3'{\cdot}2$, which even after five observations would still have an r.m.s. of $0'{\cdot}5$.[152] They would have had to recognize the periodic variation on the basis of five observations, not knowing beforehand the length of the cycle, and having to pick it out from amongst this random background. This casts severe doubt on the hypothesis on purely theoretical grounds, and demands that we examine closely any data and statistical analysis purporting to support it.

In a similar vein Hypothesis (4), although considered by the Thoms,[153] can be ruled out of court. This is because even if the periodic variation was successfully separated out, and beyond that the linear decrease of $0'{\cdot}6$ per century due to the decrease in ϵ_T, the variations of c_1 and c_2 would still appear as an effectively random perturbation completely indistinguishable from, and swamped by, the effects of (i) and (viii). Where the Thoms appear to have statistical support for this hypothesis, we must seek other explanations.

8. *Deliberate Sightlines* v. *Chance Occurrences*

We come finally in this section to question (4) of Section 1. Two methods are considered for testing the hypothesis that the forty-four sightlines were deliberate. Both involve identifying "target" differences Q from the mean standstill declinations within the lunar bands; Hypotheses (A)–(E) of Section 7 give us five possible sets of allowed targets within the lunar bands, varying in number from nine in each band down to two in the major standstill bands only.

The Thoms' approach is to consider whether the measured declination differences β cluster significantly about the targets Q. They consider only hypothesis (A) (nine targets in each band), other hypotheses having been considered with different data in earlier work.[154] In the case of their Assumption 2[155] they are effectively considering either Hypothesis (2) or (3) of Section 7; their Assumption 1 effectively considers the untenable Hypothesis (4). The Thoms' method[156] appears to amount to the following. To each sightline (measured difference β) is assigned a target difference Q, which will be that which gives the smallest residual $|\beta - Q|$ for a preconceived rough mean date of use (approximate value of ϵ_T) and mean graze effect G. From the residuals $(\beta - Q)$ obtained, mean values are calculated for northern and southern declinations. Corrected values of ϵ_T and G are then adopted so as to bring both these means to zero. However, the new values of ϵ_T and G may mean that in a few cases different targets Q will now be appropriate in order to decrease still further the $|\beta - Q|$ values concerned, and so these new targets are substituted and the whole process is repeated. This continues until a consistent set of targets, and values of ϵ_T and G, are obtained. The overall r.m.s. residual is then calculated and a probability level assigned on the basis of this.

In the first six columns of Table VIII we have reworked the Thoms' analysis with their own data. The β values listed in column 2 are deduced from the declinations given in column 5 of Table II. The targets given are those assigned by the Thoms, and the associated Q values are calculated from Table VI taking $\xi = 0$ and $\eta = 1$ (correct for Hypotheses (2) and (3)) and $\epsilon_T = 0$ and $G = 0$.

TABLE VIII. Analysis of the measured declination differences β from the forty-four sightlines and comparison with the expected, or "target", values Q.

Column headings

1 Sightline number
2 β obtained from the Thoms' surveys alone
3 Target chosen by the Thoms
4 Target value Q for $\epsilon_T = 0$ and $G = 0$
5 Residual $(\beta - Q)$ for $\epsilon_T = 0$ and $G = 0$
6 Residual $(\beta - Q)$ for $\epsilon_T = -2'\cdot3$ and $G = 0'\cdot3$
7 Residual quoted by the Thoms
8 β obtained from best available surveys
9 Target, if different
10 Target value Q for $\epsilon_T = 0$ and $G = 0$, if different
11 Residual $(\beta - Q)$ for $\epsilon_T = 0$ and $G = 0$
12 Residual $(\beta - Q)$ for $\epsilon_T = -2'\cdot1$ and $G = 0'\cdot3$

1	2	3	4	5	6	7	8	9	10	11	12
	′		′	′	′	′	′		′	′	′
1	− 5·3	$-(\epsilon+i+\Delta\ \ )$	− 7·2	+1·9	−0·7	(+0·3)	− 5·1			+2·1	−0·3
2	+ 3·0	$-(\epsilon-i\ \ \ \ )$	− 1·0	+4·0	+1·4	(+1·3)	+ 3·0			+4·0	+1·6
3	− 3·2	$-(\epsilon-i-\Delta+s)$	− 6·8	+3·6	+1·0	(+0·9)	− 2·8			+4·0	+1·6
4	−12·0	$-(\epsilon-i\ \ +s)$	−16·5	+4·5	+1·9	(+1·7)	−11·6			+4·9	+2·5
5	− 2·6	$-(\epsilon-i-\Delta+s)$	− 6·8	+4·2	+1·6	(+1·5)	− 2·8			+4·0	+1·6
6	+27·6	$-(\epsilon-i-\Delta-s)$	+24·0	+3·6	+1·0	(+0·9)	+28·4			+4·4	+2·0
7	+21·1	$+(\epsilon+i+\Delta+s)$	+24·0	−2·9	−0·9	(−0·4)	+21·1			−2·9	−1·1
8	+20·9	$+(\epsilon+i+\Delta+s)$	+24·0	−3·1	−1·1	(−0·9)	+20·7			−3·3	−1·5
9	−16·1	$+(\epsilon+i\ \ -s)$	−16·2	+0·1	+2·1	(+2·9)	−16·3			−0·1	+1·7
10	−11·7	$+(\epsilon+i+\Delta-s)$	− 6·8	−4·9	−2·9	(−2·1)	−11·7	$+(\epsilon+i-\Delta\ \ )$	−10·0	−1·7	+0·1
11	+21·8	$+(\epsilon-i+\Delta+s)$	+25·4	−3·6	−1·6	(−2·6)	+21·8			−3·6	−1·8
12	− 4·3	$-(\epsilon+i+\Delta\ \ )$	− 7·2	+2·9	+0·3	(+0·6)	− 4·3	$-(\epsilon+i-\Delta+s)$	− 6·4	+2·1	−0·3
13	−22·1	$-(\epsilon+i+\Delta+s)$	−22·6	+0·5	−2·1	(−1·9)	−22·1			+0·5	−1·9
14	− 7·1	$-(\epsilon+i+\Delta\ \ )$	− 7·2	+0·1	−2·5	(−3·0)	− 8·5	$-(\epsilon+i\ \ +s)$	−14·5	+6·0	+3·6
15	+29·1	$-(\epsilon+i-\Delta-s)$	+24·8	+4·3	+1·7	(+0·2)	+28·9			+4·1	+1·7
16	+10·5	$-(\epsilon+i+\Delta-s)$	+ 8·2	+2·3	−0·3	(−0·7)	+10·5			+2·3	−0·1
17	+12·9	$+(\epsilon+i\ \ +s)$	+14·8	−1·9	+0·1	(+0·7)	+15·9			+1·1	+2·9
18	−13·2	$-(\epsilon-i\ \ +s)$	−16·5	+3·3	+0·7	(+1·2)	−13·2			+3·3	+0·9
19	+26·6	$-(\epsilon-i-\Delta-s)$	+24·0	+2·6	0·0	(+0·1)	+27·6			+3·6	+1·2
20	− 8·1	$+(\epsilon+i+\Delta-s)$	− 6·8	−1·3	+0·7	(+1·0)	− 7·7			−0·9	+0·9
21	− 2·3	$+(\epsilon+i\ \ \ \ )$	− 0·7	−1·6	+0·4	(+0·7)	− 2·1			−1·4	+0·4
22	+14·3	$+(\epsilon+i\ \ +s)$	+14·8	−0·5	+1·5	(+1·8)	+15·3			+0·5	+2·3

1	2	3	4	5	6	7	8	9	10	11	12
	′		′	′	′	′	′		′	′	′
23	+16·1	$-(\epsilon+i\quad-s)$	+16·5	−0·4	−3·0	(−3·0)	+16·1			−0·4	−2·8
24	+ 8·9	$+(\epsilon+i+\Delta\quad)$	+ 8·6	+0·3	+2·3	(+3·0)	+ 8·9			+0·3	+2·1
25	+23·9	$+(\epsilon+i+\Delta+s)$	+24·0	−0·1	+1·9	(+2·5)	+23·9			−0·1	+1·7
26	+12·7	$+(\epsilon+i\quad+s)$	+14·8	−2·1	−0·1	(+0·1)	+12·9			−1·9	−0·1
27	−21·7	$-(\epsilon+i+\Delta+s)$	−22·6	+0·9	−1·7	(−1·8)	−23·1			−0·5	−2·9
28	+12·7	$+(\epsilon+i\quad+s)$	+14·8	−2·1	−0·1	(+0·5)	+12·3			−2·5	−0·7
29	+21·5	$+(\epsilon+i+\Delta+s)$	+24·0	−2·5	−0·5	(+0·4)	+21·5			−2·5	−0·7
30	−10·5	$+(\epsilon+i+\Delta-s)$	− 6·8	−3·7	−1·7	(−1·0)	−11·5	$+(\epsilon+i-\Delta\quad)$	−10·0	−1·5	+0·3
31	+24·9						+24·9	$-(\epsilon+i-\Delta-s)$	+24·8	+0·1	−2·3
32	−18·9	$-(\epsilon+i+\Delta+s)$	−22·6	+3·7	+1·1	(+0·8)	−18·9			+3·7	+1·3
33	−13·2	$-(\epsilon-i\quad+s)$	−16·5	+3·3	+0·7	(−0·9)	−18·0			−1·5	−3·9
34	+21·9	$+(\epsilon+i+\Delta+s)$	+24·0	−2·1	−0·1	(+0·5)	+20·7			−3·3	−1·5
35	+ 7·3	$+(\epsilon+i-\Delta+s)$	+ 5·6	+1·7	+3·7	(+1·3)	+ 6·9	$+(\epsilon+i+\Delta\quad)$	+ 8·6	−1·7	+0·1
36	+27·3	$-(\epsilon+i-\Delta-s)$	+24·8	+2·5	−0·1	(−1·6)	+23·9			−0·9	−3·3
37	−22·3	$-(\epsilon+i+\Delta+s)$	−22·6	+0·3	−2·3	(−2·5)	−21·5			+1·1	−1·3
38	+18·3	$-(\epsilon+i\quad-s)$	+16·5	+1·8	−0·8	(−1·0)	+17·7			+1·2	−1·2
39	+20·1	$-(\epsilon+i\quad-s)$	+16·5	+3·6	+1·0	(+0·5)	+20·3			+3·8	+1·4
40	+10·5	$-(\epsilon+i+\Delta-s)$	+ 8·2	+2·3	−0·3	(−0·6)	+11·1			+2·9	+0·5
41	−11·1	$+(\epsilon+i+\Delta-s)$	− 6·8	−4·3	−2·3	(−1·9)	−11·5	$+(\epsilon+i-\Delta\quad)$	−10·0	−1·5	+0·3
42	+22·8						+22·8	$+(\epsilon-i+\Delta+s)$	+25·4	−2·6	−0·8
43	−18·3	$+(\epsilon+i\quad-s)$	−16·2	−2·1	−0·1	(+0·5)	−18·3			−2·1	−0·3
44	−26·8	$+(\epsilon-i-\Delta-s)$	−22·6	−4·2	−2·2	(−1·4)	−26·8			−4·2	−2·4

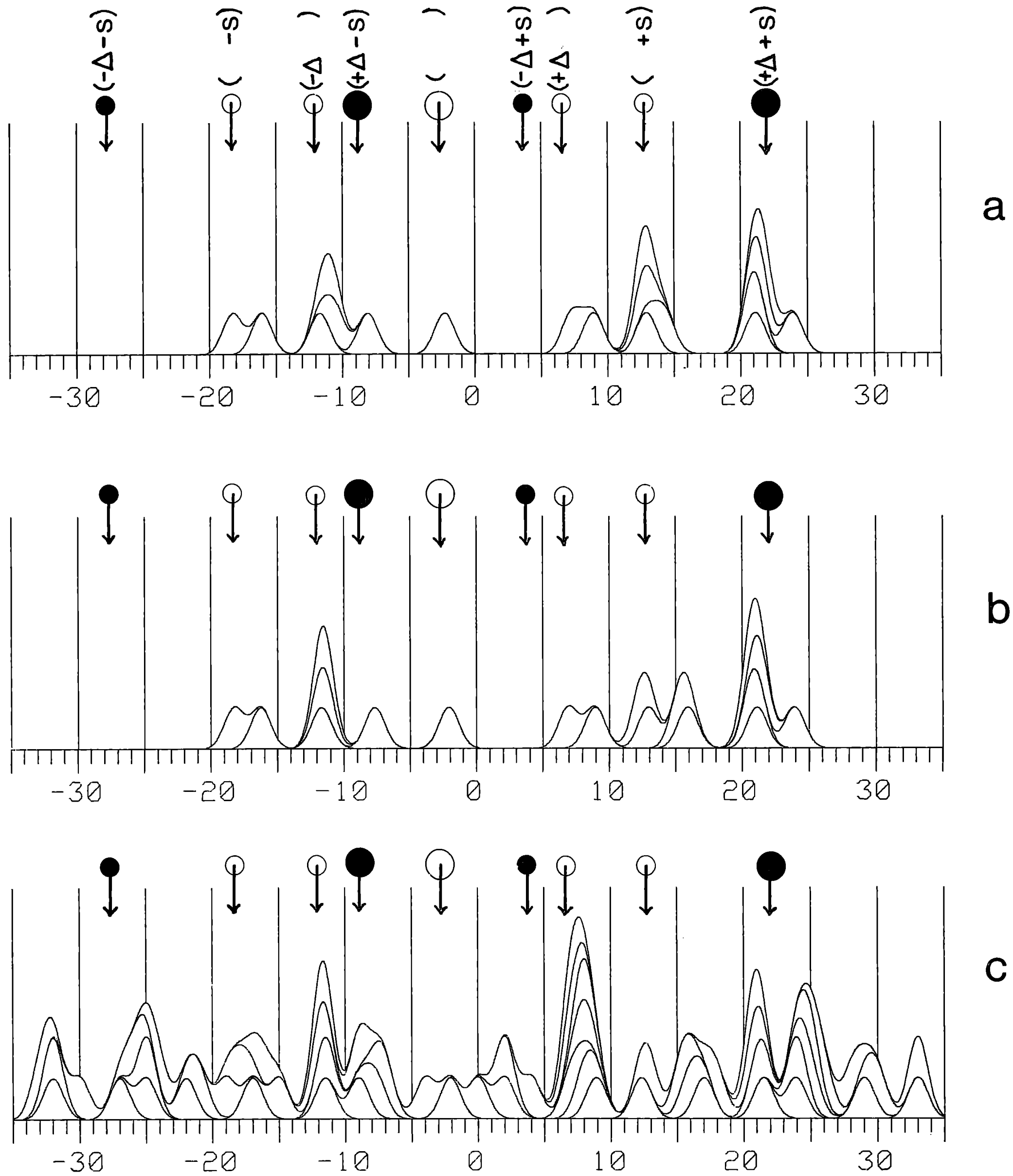

FIG. 8. β values within the $+(\epsilon+i)$ band, plotted as Gaussian probability humps of standard deviation 0′·75, for various data sets. Mean target values Q for $\epsilon_T = -2'{\cdot}3$ and $G = 0'{\cdot}3$ are marked by arrows and labelled by their nominal $\pm(\Delta \pm s)$. Circles on the arrows denote the likelihood of the target (see Section 7); large filled circles = (**) (most likely); small filled circles = ( ); small open circles = (†); large open circles = (††) (least likely).

(a) shows the Thoms' azimuth and altitude data.

(b) shows the new, more reliable, β values deduced from the data presented in Section 5.

(c) shows all equally plausible horizon features within resurveyed bands (see Section 6 and Table IX). Note that β values for features other than those included by the Thoms have been measured only to the nearest arc minute.

We obtain mean $(\beta-Q)$ values of $-2'{\cdot}0$ for northern declinations and $+2'{\cdot}5$ for southern ones; thus we take $\epsilon_T = -2'{\cdot}3$ and $G = 0'{\cdot}3$ in order to bring both means to zero. The new $(\beta-Q)$ values obtained are listed in column 6, for comparison with those obtained by the Thoms themselves, which are listed in column 7. The values differ by $2'{\cdot}4$ at line 35, up to $1'{\cdot}6$ at three other lines, and up to $1'{\cdot}0$ at the rest. This is presumably largely due to the Thoms' varying corrections for graze from line to line,[157] but will also partly be due to our approximations (see Section 7) to eliminate site-dependent terms. However, there is no evident systematic difference between our values and the Thoms', and we obtain an r.m.s. residual of $1'{\cdot}55$, in line with the value of $1'{\cdot}52$ obtained from the residuals given by the Thoms. The mean date we deduce, corresponding to $\epsilon_T = -2'{\cdot}3$, is about 1650 B.C., again in line with the Thoms' result.

The probability estimate we obtain will depend crucially upon where we take the edges of the lunar bands, the appropriate limits being determined by the selection of horizon features for inclusion in the first place. Judging by the limits of their histograms[158] the Thoms seem to have considered a band width of around 58′ to 60′ when selecting features for inclusion, but their probability estimate using Broadbent's criterion[159] may be misleading. It depends critically upon the assumed average spacing between targets, which the Thoms take as 8′, whereas in fact the targets are irregularly spaced.

Consider instead points (or β values) distributed randomly, *i.e.* uniformly, along the $+(\epsilon+i)$ band between $-30'$ and $+30'$. Given that the assumed target Q is always that nearest any given β, we obtain an expected value for $|\beta-Q|$ of $1'{\cdot}90$ and an r.m.s. residual of $2'{\cdot}31$. Contracting the band to only 3′ beyond the outermost targets (*i.e.* $-28'{\cdot}6$ to $+27'{\cdot}0$ for $\epsilon_T = 0$ and $G = 0$) the expected $|\beta-Q|$ becomes $1'{\cdot}72$ and the r.m.s. residual $2'{\cdot}06$. (Values will be almost identical for the other lunar bands.) Taking the value obtained above for the measured r.m.s. residual ($1'{\cdot}55$) and performing a χ^2 test, we find the data to be significant at the 0·1 per cent level (*i.e.* highly significant) for the 60′ band width, but only at a 1·2 per cent level (*i.e.* marginally significant) for the band width extending just 3′ beyond the outermost targets. In Figure 8 (a) we have plotted the β values for the $+(\epsilon+i)$ band, which contains eighteen out of the total data set of forty-two lines, in the Thom style of a histogram of Gaussian probability humps for direct comparison with the top quarter of the Thoms' histogram.[160] Both our measured and target values are shifted to the left with respect to the Thoms' diagram, because we have referred both to the standard epoch of 2000 B.C. We have taken the standard deviation of each hump to be $0'{\cdot}75$, which by inspection gives roughly the same spread as the Thoms use. We note that the Thoms omitted three of the nine targets (the solstitial ones) from their diagram, which, as only one of the eighteen β values happens to fall nearest a solstitial target, exaggerates the apparent clustering of the humps around the remaining targets. As we noted in the previous section, the averaging of solstitial and equinoctial observations would require existing observations of both, and so it would surprise us if the shunning of solstitial markers in favour of equinoctial and averaged ones in roughly equal numbers were a real effect.

In columns 8–12 of Table VIII we have reworked the analysis using the values of declinations for the forty-four sightlines now considered more reliable, as given in column 14 of Table II. After choosing the best targets for $\epsilon_T = -2'{\cdot}3$

and $G = 0'{\cdot}3$, and then twice performing the iterative process described at the beginning of this section, we obtain a consistent set of targets with mean residuals $(\beta - Q)$ of $-1'{\cdot}7$ for northern declinations and $+2'{\cdot}4$ for southern ones. Thus we take $\epsilon_T = -2'{\cdot}1$ and $G = 0'{\cdot}3$ in order to bring both means to zero, and we obtain finally an r.m.s. residual of $1'{\cdot}76$. For the $60'$ band width this is significant at a 1·3 per cent level (*i.e.* marginally significant), but for the band width extending just $3'$ beyond the outermost targets the significance level rises to 9·9 per cent and the data are no longer significant. In Figure 8 (b) we have plotted a histogram for the new β values within the $+(\epsilon+i)$ band.

Clearly the apparent significance of the data is strongly dependent upon the assumed band width, and in the absence of data on selection criteria we can only quote upper and lower bounds for the true figure. The substitution of new β values considered more reliable in place of the Thoms' values alone has the effect of decreasing the significance, and the convergence process used to obtain optimal values of ϵ_T and G in the absence of prior knowledge about them will further decrease the true figure, possibly considerably. Finally, we have taken no account of the selection of horizon features as putative foresights well within the lunar bands. It is possible to repeat the entire analysis using, instead of just the foresights proposed by the Thoms, all the equally plausible horizon features identified in Section 6 (see Table IV). If this is done, all evidence in favour of deliberate clustering around the nine targets within each band, as well as any evidence of preferential clustering around targets other than solstitial ones, entirely disappears. This is evident in Figure 8 (c), where we have included all other features within those $+(\epsilon+i)$ bands which were successfully resurveyed.

In the discussion so far we have taken no account of the unavoidable uncertainties in the Q values given in Table VII. Under Hypothesis (2) the r.m.s. residual in the Thoms' data is well below the unavoidable r.m.s. variation in the targets themselves; likewise the case of Hypothesis (3) with a span of 1000 years in sightline construction dates. Thus we are forced to accept the Thoms' data as evidence for Hypothesis (3) with a span of at most 500 years in construction dates, or else to demonstrate an alternative explanation in terms of selection effects in the data. In view of the comments above and of the unlikelihood of Hypothesis (3) on purely theoretical grounds (see Section 7) we find the evidence overwhelmingly against the first option.

We now suggest an alternative statistical test which takes into account both the inherent uncertainties in the measured declinations and the unavoidable variations in the target values. It will also enable us to consider whether the data provide evidence for lunar observations at any level of precision. Each measurement, given the inherent uncertainties in the observing position and the inevitable presence of measurement errors (see Section 5 for details of both), produces a range of β values which might have been intended if the sightline was intentional. Each combination of Hypotheses (1)–(3), (A)–(E) and postulated span of construction dates produces a set of mean Q values each with an associated r.m.s. variation. We shall consider a measurement to score a "hit" upon a target if any part of the range of β values from the measurement falls within one r.m.s. variation of a mean target value Q. Under any particular hypothesis we then test whether the number of hits is significantly greater than would have been expected by chance. Our test statistic will be that z derived by Freeman and

Elmore.[161] We assume (ignoring variations in $|\partial\delta/\partial A|$ from sightline to sightline) that a random distribution of points in azimuth will produce a uniform distribution of β values within each lunar band. The probability p_i of a chance hit then reduces to the probability that a block of β values equal in width to thal produced by the measurement, when randomly placed within a lunar band, wilt overlap one of the target zones. All background information necessary for performing this test has been given in Section 7.

In what follows we shall consider only a simplified version of this analysis, by way of illustration and as a pointer for future work. Instead of regarding each lunar band as a continuous range, we quantize it into 71 one-minute bins running from $-35'{\cdot}5$ to $-34'{\cdot}5$, $-34'{\cdot}5$ to $-33'{\cdot}5$ and so on up to $+35'{\cdot}5$. Under any hypothesis each target zone occupies a certain number of bins, so each will either represent a hit or not. We simplify still further by assuming that each measurement occupies exactly one bin. Since under any of the available hypotheses the number of bins representing hits in each of the four bands will differ by at most one or two from a mean value m, we may take all the p_i equal to the constant $p = m/71$, and our test statistic reduces to the familiar binomial expression

$$z = \frac{r - np}{[np\,(1-p)]^{1/2}}$$

where r is the number of hits and n is the total number of measurements. (Note that this z now has the opposite sign to Freeman and Elmore's.)

In Table IX we have listed β values to the nearest minute for each of the horizon features identified in Section 6. We consider them to be fairly selected, and to be exhaustive within the band limits $-35'{\cdot}5 < \beta < +35'{\cdot}5$ except in two cases where measurements were not taken out to these limits (see Table IV). The organisation of Table IX suggests ways in which feature type, efficacy of the indication, and archaeological status of the sightline may be taken into account by considering subsets of the total data set. In testing each hypothesis, we have considered four cases: (a) all features are included regardless of status; (b) all features are included except those at lines of status X, Y or Z (*i.e.* non-indicated horizons are excluded together with sites ruled out on archaeological grounds—see Section 4); (c) only features falling within an IAR or AAR at a line of archaeological status A are included (*i.e.* only sightlines of reasonable archaeological status and only those features falling within 5° of the bounds of an indication remaining today); and (d) only features falling within an IAR at a line of archaeological status A are included (*i.e.* only lines of reasonable archaeological status and only those features falling within the bounds of an indication remaining today). Feature types I–V represent, in an intuitive sense, an ordering of features in terms of their effectiveness if used as foresights. Thus in each of cases (a)–(d) we consider five sub-cases; including all feature types, omitting Type V, omitting Types IV and V, and so on.

We tested each of Hypotheses (1)–(3) in combination with each of Hypotheses (A)–(E) for an assumed span of construction dates of 500 years. In the case of Hypothesis (3) we tried ϵ_T values of 0 and $\pm 3'$, corresponding to mean dates of about 2500 B.C., 2000 B.C. and 1500 B.C., but for Hypotheses (2) and (1), which give much wider target zones, we tried only $\epsilon_T = 0$. We always assumed $G = 0$. Only in the case of Hypothesis (2) did we obtain any z values substantially

TABLE IX. β values in minutes for equally plausible horizon features within 33 lunar bands containing proposed foresights. They are organized by feature type, whether the feature concerned falls within an IAR or AAR (see Section 6), and by the archaeological status of the line. Figures in parentheses give the sightline number concerned, and * denotes features included by the Thoms as proposed foresights.

		$+(\varepsilon+i)$	$+(\varepsilon-i)$
IAR, status A			
	Type I	+12* (28)	
	Type II	−32 (28), −27 (20), −12* (30), −9 (21), −2* (21), +9* (24), +17 (28), +22* (28)	
	Type III	−32 (20), −25 (28), −17 (28), +8 (21), +16* (17), +33 (28)	−35 (42), −29 (42), −20 (42), −10 (42), −6 (42), −4 (42)
	Type IV	+24* (24), +29 (28)	
	Type V	0 (17), +24 (28)	
AAR, status A			
	Type I	−12* (41), +7 (30), +30 (24)	
	Type II	−30 (30), −22 (26), −21 (21), −15 (20), −8* (20), −7 (30), −4 (22), +2 (30), +8 (22), +8 (26), +15* (22), +21* (7), +25 (7), +25 (30)	+23* (42)
	Type III	−33 (41), −26 (21), −24 (41), −12 (26), −9 (22), +2 (20), +4 (41), +13* (26), +18 (30), +25 (22)	
	Type IV		
	Type V	−19 (17)	
IAR and AAR, status B and C			
	Type I		
	Type II	+18 (8), +21* (8)	
	Type III		
	Type V	−16* (9), −12* (10)	
Status X, Y (unindicated) and Z			
	Type I	+26 (34), +28 (35)	
	Type II	+7* (35), +21* (34), +33 (35)	+22* (11)
	Type III		−22 (11), +14 (11)
	Type IV	+7 (35)	
	Type V		

$-(\varepsilon-i)$	$-(\varepsilon+i)$
	−16 (23), −14 (23)
−31 (19), −27 (19), −18 (19), −1 (19), 0 (19), +7 (19)	−17 (27)
+12 (19)	
	−17 (23), −6 (23)
−9 (19), +28* (6), +28* (19), +31 (19)	
−13 (2), −8 (2), −7 (2), +3* (2), +3 (2)	−24 (27), −23* (27), +16* (23), +24 (23)
−6 (2)	
−2 (2)	
	−17 (27), +31 (23)
−3* (5)	
	+24* (36), +31 (15)
	+ 5 (36), +29* (15)
−18* (33)	
−28 (4), −22 (4), −21 (4), −19 (3), −13 (3), −12 (3), −12* (4), −12 (4), −3* (3), −3 (3)	−34 (39), −28 (14), −24 (14), −19* (32), −13 (14), −13 (32), −12 (14), −11 (14), −10 (32), −7 (14), −5* (1), −4 (32), −4 (39), +6 (32), +6 (39), +8 (39), +11* (39), +18* (37), +19 (32), +27 (32)
−20 (4), −12 (3)	−22* (37), −21 (32), −16 (32), −9 (32), −4 (39), −1 (39), +5 (37), +12 (37)
	+14 (1), +18 (37)
−16 (4), −7 (3)	−25 (39), −23 (37), +1 (14)
	− 9* (14), −3 (39), 0 (14), +1 (32), +8 (14), +13 (39), +20* (39), +22 (1)
	−17 (39), +9 (39), +14 (39), +16 (37)

TABLE X. Results of the "hit or miss" statistical test for hypothesis (2) and various subsets of the total data set of horizon features described in the text.

		2A ($p = 47/71$)			2B ($p = 39/71$)			2C ($p = 19/71$)			2D ($p = 8/71$)			2E ($p = 8/71$)		
		n	r	z	n	r	z	n	r	z	n	r	z	n	r	z
All features	All types	161	128	3·6	161	108	3·1	161	55	2·1	161	26	2·0	117	21	2·3
	I–IV	147	118	3·6	147	100	3·2	147	53	2·5	147	24	1·9	108	19	2·1
	I–III	132	105	3·2	132	90	3·1	132	48	2·5	132	22	2·0	93	17	2·1
	I–II	100	80	2·9	100	68	2·6	100	37	2·3	100	18	2·1	74	15	2·4
	I only	49	40	2·3	49	36	2·6	49	19	1·9	49	9	1·6	34	7	1·7
Status X, Y and Z excluded	All types	93	71	2·1	93	62	2·3	93	34	2·1	93	14	1·2	66	11	1·4
	I–IV	83	65	2·3	83	58	2·7	83	33	2·7	83	13	1·3	61	10	1·3
	I–III	77	61	2·4	77	54	2·7	77	31	2·7	77	12	1·2	55	9	1·2
	I–II	52	42	2·2	52	37	2·4	52	23	2·8	52	11	2·3	39	8	1·8
	I only	17	15	1·9	17	15	2·8	17	9	2·4	17	4	1·6	12	2	<0
IAR and AAR, arch. status A only	All types	84	65	2·2	84	56	2·2	84	32	2·3	84	13	1·2	58	10	1·4
	I–IV	76	61	2·6	76	54	2·8	76	31	2·8	76	12	1·2	55	9	1·2
	I–III	70	57	2·7	70	50	2·8	70	29	2·8	70	11	1·2	49	8	1·1
	I–II	46	39	2·7	46	34	2·6	46	21	2·9	46	10	2·2	33	7	1·8
	I only	15	14	2·2	15	14	3·0	15	8	2·3	15	4	1·9	10	2	0·9
IAR, arch. status A only	All types	41	30	0·9	41	26	1·1	41	12	0·4	41	3	<0	24	3	0·2
	I–IV	35	27	1·4	35	24	1·6	35	11	0·6	35	2	<0	22	2	<0
	I–III	31	24	1·3	31	21	1·4	31	9	0·3	31	1	<0	18	1	<0
	I–II	18	16	2·0	18	13	1·5	18	5	0·1	18	1	<0	12	1	<0
	I only	3	3	1·2	3	3	1·6	3	0	<0	3	0	<0	3	0	<0

greater than 2; so in the other cases we can safely conclude that there is no evidence from these data for the lunar hypothesis.[162]

We have listed the results in full for Hypothesis (2) in Table X. We require *z* values greater than about 3 in order to provide clear evidence of trends deviating from chance occurrences. Values above 3 occur when testing Hypotheses (2A) and (2B) with features of every status and all or most types included. However under these hypotheses the target zones overlap considerably and fill most of the central parts of the lunar bands (more than about 10′ from their limits). Thus the high *z* values here may be doing no more than reflecting a general concentration of features away from the very edges of the bands, rather than providing evidence of precision observations of any of the eight or nine lunar events within each band. This is borne out by the fact that the data include from Lines 1 and 39/40 some seventeen features strongly concentrated towards the band centres owing to the lack of measurements out to the azimuth limits of the band. Both these lines are of status Y, and when they are excluded along with the other lines of status X, Y or Z, the *z* values drop considerably. Nonetheless they are still generally well above 2, and so there still remains some evidence of concentration of features away from the (target-free) band edges. We are unable to explain this away by selection effects at the sites considered.

Of greatest interest are the *z* values of around 2·8 obtained from Hypothesis (2C) when IAR and AAR features are included, regardless of whether or not lower archaeological status (B and C) lines are included. Under this hypothesis each lunar band contains only four well-separated target zones representing directly observable lunar events; thus we appear to have marginal evidence for sightlines recording these events, using extrapolation but set up after a single occurrence of the event in question. The *z* values drop markedly when the AAR features are excluded, suggesting that the indications remaining today are generally accurate to no more than about 5° in azimuth. However, before we can accept this as reliable evidence several checks are necessary. We need to know the effects upon the results of changing our hypothesis more finely; whether any other effects might cause the observed trend; whether the horizon features contributing to the trend possess any common characteristics; whether the sites contributing to the trend have any archaeological coherence; and whether the marginal significance is increased or disappears completely when data from a wider selection of sites are added. Such work is in progress.

9. *General Conclusions and Comments*

At the beginning of Part One of this paper we posed four key questions pertinent to a reassessment of the high precision lunar sightlines. Answers can now be summarized.

(1) Of the forty-four putative sightlines, only fourteen represent cases where structures remaining today accurately indicate the proposed foresight. Of these, three seem somewhat dubious and one very dubious on archaeological grounds, and the remaining sites do not manifest any obvious archaeological coherence. In a further fourteen cases the azimuth range indicated misses the proposed foresight by up to 5°; in other words

the indication, if deliberate, was only rough. Thirteen of the remaining foresights are entirely unindicated, although two (at Brogar) are indicated from another position from which they are also postulated to have been observed. The last three sightlines can be ruled out as intentional. Rather than dismissing various sightlines out of hand, or else including them all indiscriminately, we prefer to go through any analysis several times using various subsets of the data corresponding to more or less stringent exclusion criteria.

(2) We have been able to visit all the relevant sites and to fill in backsight and indication information not given by the Thoms. Resurveys of thirty-eight of the forty-four sightlines produced general agreement with the Thoms' declinations to within about 1′. Only in two cases did two surveys, both considered reliable, produce differences substantially greater than this (up to 4′), and in one of these our values were in good agreement with alternative values supplied elsewhere by the Thoms. Comparison of surveys considered reliable confirms the contentions of other authors that measured declinations are in general only reliable to about 1′.

(3) Using a classification system for all horizon features based upon that of the Thoms for assumed foresights, we have uncovered a total of 161 horizon features, equally plausible *per se* as foresights, from within lunar bands containing thirty-seven of the foresights listed by the Thoms. We have organized them by type under the classification system, and also by whether they fall within an indicated azimuth range, adjacent to (*i.e.* within 5° of) such a range, or neither. If we are to be sure that our data are free of subjective bias, we must use in any analysis all of these horizon features, or else clearly-defined subsets based only upon feature type or state of indication.

(4) We have reworked the Thoms' analysis of their own data in a way which makes all the assumptions explicit. We find that the very high significance they obtain will be exaggerated because of a misleading assumption in the statistical test they use; and in any case it depends critically upon factors such as the assumed width of the lunar bands. It should also be reduced, possibly considerably, on account of the unknown parameters in the "expected" declinations which can be adjusted until they fit the data best. When the Thoms' data are replaced by new, more reliable, values based upon both their surveys and the author's, the significance is reduced still further, and, depending upon the band width which is in fact appropriate (in view of the Thoms' (unknown) criteria for selecting foresights), may become marginal or disappear completely. When other, equally plausible, horizon features are included in the analysis all evidence for lunar observations to a precision of one or two minutes of arc, obtained by averaging measurements taken over periods of up to 200 years, totally disappears. Since there are persuasive theoretical arguments as to why such an observation programme would in practice have been almost if not completely impossible, this result is perhaps not surprising. A different statistical technique from the Thoms', taking into account inherent

uncertainties in the measured declinations and in the theoretical targets (which themselves move about), was applied to the more objective data set consisting of all horizon features equally plausible *per se* as foresights. Marginally significant evidence was obtained in support of the recording of the "directly observable" lunar events $\pm(\epsilon \pm i \pm \Delta \pm s)$ (as opposed to events only recordable by averaging observations of other events) to a precision of about 5′, using foresights indicated roughly (*i.e.* to within about 5°). Before any reliability can be placed upon this evidence, though, more detailed analysis is necessary, and more data are required in order to answer questions about the selection of sites and lunar bands themselves. Such work is under way as part of a project to reassess Level 1 of the Thoms' cumulative structure of evidence in favour of megalithic astronomical sightlines.

Our conclusions have been very different from those of the Thoms, yet based upon data from the same sites. The major cause of the discrepancy lies in the Thoms' selection of putative sightlines. We know that the Thoms *have* selected both observing sites and horizon features on the basis of astronomical considerations. A. S. Thom writes:[163] "If we stand at a marked backsight and make careful observations of the profile of part of the horizon which turns out to contain a significant declination we can assume that we are at a real observing point." In other words if the declination does not turn out to be significant, the site is dismissed as a possible observing position. Then again:[164] "If there is a notch or mark or a clearly defined foresight of any kind on the profile then its declination must be calculated, and if it turns out to be a pertinent declination then we know that we are at a megalithic backsight." In other words if the declination does not turn out to be pertinent, the horizon feature is ignored and not included in any statistical analysis. Although the Thoms felt quite justified in proceeding in this way, the results of any statistical analysis are surely worthless unless the data can be shown to have been selected fairly, that is *without regard for the astronomical possibilities*. Whether this is achieved by using pre-defined selection criteria (a procedure favoured by the author) or otherwise, demonstrably fair selection of data is of paramount importance.

Acknowledgements

I am most grateful to Douglas Heggie and Leslie Morrison for criticism of an earlier draft of Section 7 of this paper, and to Peter Freeman for helpful discussions on the content of Section 8. My thanks are due to the following, who assisted me in the surveying work during 1979 and 1981 at the relevant sites: Clare Huxley, Sal Brown, Mari Williams, Mary Wilson, Phil Appleton, Kim Bramley, Sheila Brookes, Chris Jennings, and Ray and Cilla Norris. Kim Bramley also helped with great efficiency in the reduction of the survey data. The Ordnance Survey were kind enough to let me have access to essential triangulation station information at only a nominal charge. In addition to the grant foundations listed in Part One (Science and Engineering Research Council, University of Wales and the Leverhulme Trust), a grant from the British Academy towards the reduction work is gratefully acknowledged.

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90. L. V. Morrison, "On the analysis of megalithic lunar sightlines in Scotland", *Archaeoastronomy*, no. 2 (1980), S65–77, p. S73.
91. Declinations quoted in *AA2* and *TBAR* assume a horizontal parallax of 56′·4 or 57′·4, and various temperatures and pressures, according to the position of the site and lunar event assumed (for two typical calculations shown in full, see *JHA*, 175). These declinations typically differ from our mean values by between 0′·5 and 1′·0, the largest difference being 1′·2 at Callanish (Line 14). Declinations quoted in *MLO*, 76 agree with our values to within 0′·4, except for a 0′·9 difference in the case of Corogle Burn (Line 16).
92. *MLO*, 76.
93. *MRBB*, 134.
94. Using the formula for rate of change of declination with azimuth

$$\left|\frac{\partial\delta}{\partial A}\right| = \frac{\cos\phi\cos h\sin A}{\cos\delta} \qquad \text{(see } MLO\text{, 27)}$$

for latitude ϕ between 52° and 60° and small altitude h, we obtain for minor standstill ($|\delta| \approx 19°$ and $|\sin A| \approx 0{\cdot}8$) $|\partial\delta/\partial A| \approx 0{\cdot}4$; and for major standstill ($|\delta| \approx 29°$ and $0 \lesssim |\sin A| \lesssim 0{\cdot}7$) $0 \lesssim |\partial\delta/\partial A| \lesssim 0{\cdot}4$ with a most probable value around 0·3.
95. *JHA*, 176, Table 3.
96. J. A. Cooke, R. W. Few, J. G. Morgan and C. L. N. Ruggles, "Indicated declinations at the Callanish megalithic sites", *Journal for the history of astronomy*, viii (1977), 113–33; see pp. 117–18 for details of surveying technique.
97. See *A map of the standing stones and circles at Callanish, Isle of Lewis, with a detailed plan of each site* (Glasgow, 1978), produced by the Department of Geography, University of Glasgow, under the direction of Dr D. A. Tait.
98. *MLO*, 29–32.
99. *MLO*, 93.
100. *MLO*, 75.
101. As deduced from the term 0·01 D (K_n–K_o) in Thom's formula (3·9) (*MLO*, 32), taking the maximum variation in K as 8, in line with *MLO*, 30.
102. J. D. Patrick, "A reassessment of the lunar observatory hypothesis for the Kilmartin stones", *Archaeoastronomy*, no. 1 (1979), S78–85, p. S82.
103. *MRBB*, 125.
104. *MRBB*, 124, Fig. 10.3.
105. *MLO*, 68; see *MLO*, 123 for the method.
106. *MLO*, 56.
107. *MLO*, 48.
108. *MLO*, 66.
109. *MLO*, 59.
110. *MLO*, 73.
111. For a site plan see *MLO*, 71.
112. *MLO*, 71 and 72.
113. *MRBB*, 126; for a profile diagram see *MRBB*, 124, Fig. 10.4.
114. *MRBB*, 173.
115. *AA2*, S81; *TBAR*, 32–33.
116. Compare Thom's diagram, *MLO*, 46, inset (a).
117. See *MLO*, 65, Fig. 6.10.
118. *MLO*, 57.
119. *MLO*, 60, Fig. 6.2.
120. For a site plan and nomenclature see *MLO*, 46.
121. For a site plan and nomenclature see M. R. Ponting and G. H. Ponting, "Decoding the Callanish complex—some initial results", in *BAR*, 63–110, p. 80.
122. See Thom's site plan, *MLO*, 94.
123. A. Thom and A. S. Thom, "Astronomical foresights used by Megalithic Man", *Archaeoastronomy*, no. 2 (1980), S90–94.
124. See *RBAR*, Section 4.3, where it is reassessed as part of the Level 2 (lower precision) hypothesis not involving horizon foresights.

125. *RBAR*, 183; see also T. McCreery, "Megalithic lunar observatories—a critique, Part I", *Kronos*, v (1) (1979), 47–63.
126. Thom and Thom, *op. cit.* (ref. 123), S93, Table 1.
127. *MRBB*, 124, Fig. 10.2.
128. *TBAR*, 24.
129. *MLO*, 39.
130. Morrison, *op. cit.* (ref. 90).
131. See, *e.g.*, *AA2*.
132. Morrison, *op. cit.* (ref. 90), S67 and S72.
133. Morrison, *op. cit.* (ref. 90), S67–69 and S72; note that his "Δ" sometimes differs from the Thoms' in sign, and also that it includes a term $d\delta_2$ (equivalent to the Thoms' c_2) which, following the Thoms, we consider separately below.
134. Morrison, *op. cit.* (ref. 90), S74.
135. Morrison, *op. cit.* (ref. 90), S73–74.
136. See *JHA*, 174 for an explanation, and *AA2*, S86–87 for latest estimates of the value of the graze.
137. Morrison, *op. cit.* (ref. 90), S67.
138. Morrison, *op. cit.* (ref. 90), S69–72.
139. *MLO*, ch. 8.
140. *AA2*, S85–86. The Thoms' c_1 and c_2 are equivalent respectively to Morrison's $d\delta_1$ and $d\delta_2$ (*op. cit.* (ref. 90), S69).
141. Morrison, *op. cit.* (ref. 90), S73. In calculating dp, Morrison took $\phi = 57°$; even for Parc-y-Meirw ($\phi = 52°$) an error of only about $0'{\cdot}01$ is introduced.
142. In *JHA*, 175, the Thoms consider the alternative case and give it equal weight, but in the analysis of the 42 lines, the possibility of eliminating some less likely alternatives is included in the analysis (*AA2*, S86–87)..
143. These values are not dissimilar to those assumed by the Thoms (see, *e.g.*, *JHA*, 175). We find that differences in mean barometric pressure for different times of year and for different heights of site are negligible.
144. Larger errors are to be expected for one or two foresights at altitudes below 0°, but here the refraction correction for given conditions itself becomes much more uncertain anyway.
145. Morrison, *op. cit.* (ref. 90), S69. The variation is produced by a complex interaction of terms: see J. M. A. Danby, *Fundamentals of celestial mechanics* (New York, 1962), 282.
146. Morrison, *op. cit.* (ref. 90), S74. For a sinusoidal variation, the r.m.s. is $1/\sqrt{2}$ times the amplitude of the variation.
147. Morrison, *op. cit.* (ref. 90), S74. Up to $\pm 0'{\cdot}2$ of the variation in both cases is sinusoidal with period half of 179 years, but this is a negligible perturbation on the main variation. Variations in altitude given by Morrison's formulae have been multiplied by 0·925 to bring them to changes in declination.
148. Maximum values are deduced from the formula for ϵ_T; r.m.s. values are deduced by assuming a uniform distribution over the given dates, hence they are $1/\sqrt{3}$ times the maximum variation.
149. From Bessel's formulae (ref. 89), a variation of, say, ± 20 mb at 10°C gives a variation in the altitude correction of about $\pm 0'{\cdot}8$ at $h = 0°$, $\pm 0'{\cdot}6$ at $h = 1°$ and $\pm 0'{\cdot}2$ at $h = 5°$; say $\pm 0'{\cdot}5$ on average. A variation of, say, ± 10°C at 1005 mb gives a variation of about $\pm 0'{\cdot}8$ on average. Thus we estimate a possible overall variation of up to $\pm 1'{\cdot}3$ in altitude, or $\pm 1'{\cdot}2$ in declination. The r.m.s. value is estimated by assuming a uniform distribution over this range.
150. Fluctuations under (ii) and (iii) are related, running in phase for upper limb observations and out of phase for lower limb ones: see Morrison, *op. cit.* (ref. 90), S74. The other variations are mutually independent.
151. At about five of the ten relevant standstills occurring during one cycle, any event will occur at the alternative possible month and time of day and hence be unobservable. Even observing all five possible occurrences, given the need for extrapolation, requires runs of good weather just at the appropriate times. For a fuller discussion see *RBAR*, 194–5 and references therein.
152. Combining terms (i), (vii) and (viii) gives a variation of up to $\pm 2'{\cdot}6$ with an r.m.s. of $0'{\cdot}9$, which reduces after five observations to $0'{\cdot}4$. In addition, the (almost) linear decrease in ϵ_T over 179 years leads (unless it is recognized as such) to a further effective variation of $\pm 0'{\cdot}6$ with r.m.s. $0'{\cdot}3$. Hence the total quoted. The quoted uncertainties

in Q in Table VII include, in addition to this, the uncertainty in term (v) due to our lack of knowledge about the exact date of construction of any sightline.

153. See, *e.g.*, *AA2*, S85–88. Hypothesis (4) corresponds to the case $c_S = 0$.
154. *MSB*, ch. 10; *MLO*, ch. 7.
155. *AA2*, S81, Table 1.
156. *AA2*, S83.
157. That varying corrections for graze from line to line *have* been employed is clear from the phrase (our italics) "The value of the [mean] graze, *or if we have already used a graze, the correction to this*, is then given by . . ." (*AA2*, S83).
158. *JHA*, 178, Fig. 4; *AA2*, S82, Fig. 2.
159. *AA2*, S84.
160. *AA2*, S82, Fig. 2.
161. P. R. Freeman and W. Elmore, "A test for the significance of astronomical alignments", *Archaeoastronomy*, no. 1 (1979), S86–96, p. S89.
162. *Ibid.*, S89–90.
163. *TBAR*, 19.
164. *TBAR*, 24.